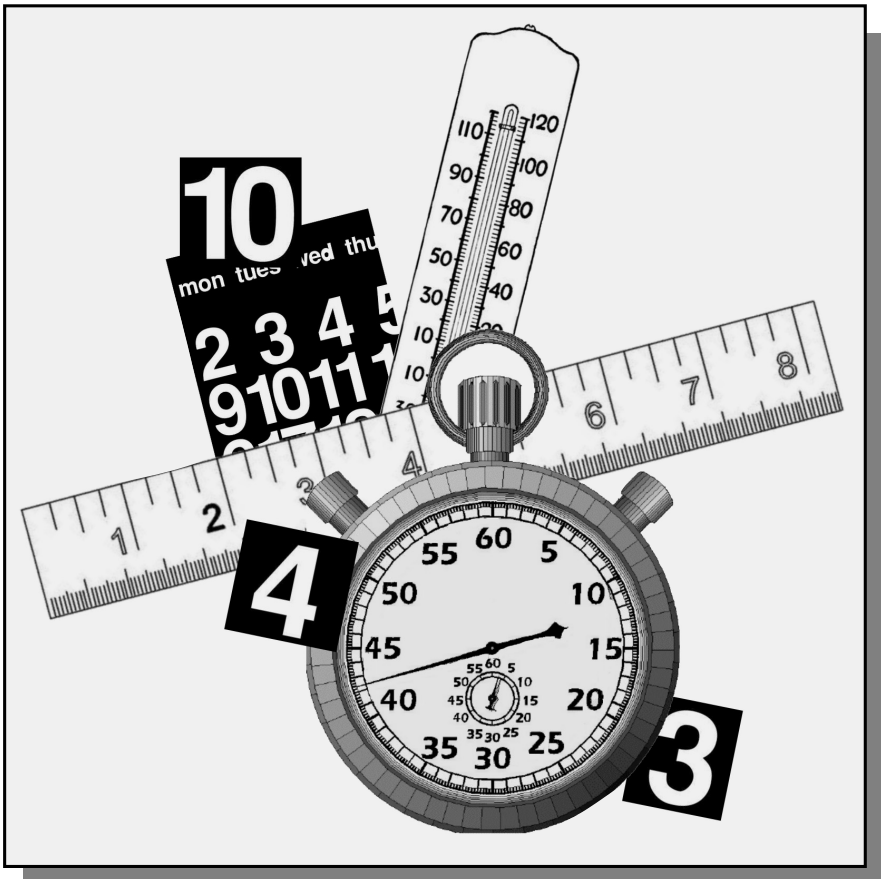
# *Basic Math Skills*




## *Chapter 1*

BMS: 76023 Revision: 030208

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

# PROFESSIONAL CREDITS

## *Basic Math Skills*

### *Chapter 1*

**Publisher and Managing Editor**
Ronald E. Johnson, Ph.D.

**Writers**
Kermit Northcutt
David Browder

**Technical Editor**
David Browder

**Content Editor / QA**
Nancy Johnson

**Production Manager**
H. Michael Young

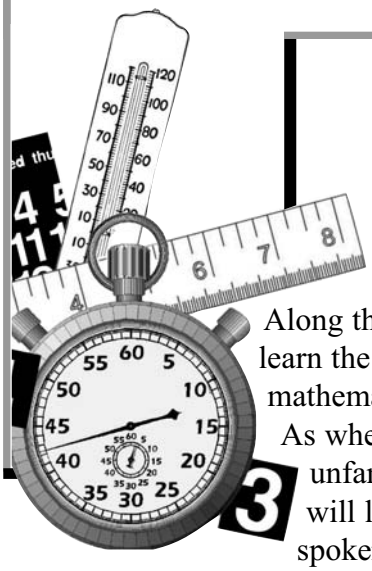
**Page Composition**
Debbie L. Dial
Kathy L. Dial
Pamela K. Hurst
H. Michael Young

**Graphic Artists**
Kathy L. Dial
Pamela K. Hurst
Kristin Odom
Lydia Seals
Don Ward
Dan White
H. Michael Young



©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

# *Basic Math Skills*
# INTRODUCTION



You are about to take a journey through the land of Mathematics with Professor K. Along the way you will learn the history of many mathematical concepts. As when going to any unfamiliar land you will learn the language spoken there and how to use the tools of that land to accomplish the job set before you.

All of us use mathematics in some form or another everyday, from currency exchange to weighing and measuring. Math can be fun if taken one small step at a time. You have probably solved riddles just for fun. The fun in a riddle is in the solving of it, the satisfaction of getting to the right answer. Math can be fun too, because you are given a problem and challenged to find the answer. As with riddles, problems can be fun in the quest for solutions. You can have fun as Professor K reviews mathematical concepts you already know and shows you new ones.

Since the dawning of civilization man has seen the importance of mathematics. Mathematics began as simply tallying or counting possessions, such as camels, sheep, or pieces of gold or silver. Man has applied math skills in trading, keeping up with the passage of time, and inscribing geometric shapes on pottery and patterns woven into cloth.

You probably remember from history, stories of the Egyptian pyramids. Ancient architects could not have built the pyramids without the use of mathematics. Even today we do not know how men built pyramids in Egypt, Mexico, Cambodia and elsewhere, without the use of today's technology and machinery. Mathematics and physics enabled ancient cultures to build geometric wonders to precision.

Ancient Greeks, Romans, Aztecs and Incas used mathematics to explain the universe in light of religion. Ancient cultures in Central and South America, France, England, Cambodia and Egypt used similar mathematics to plot orbits of stars and to duplicate the constellations dating back as far as 10,500 years ago. You may recall hearing the names of such men as Archimedes, Plato, Newton, and Einstein. These are just a few of the men who have contributed to what we know today about mathematics. Some of these men saw and solved problems just for fun. Others diligently worked for years not giving up just because the answer was not easy to identify. You, too, can do the same and make math an enjoyable educational experience that can last your entire life.

You remember from history books the stories of early explorers and the founding of America. Writers used time lines to give you an idea of when events took place in relationship to other events. When looking at time lines, you see only key years displayed. Some time lines have gaps of many years between key dates. To find the number of years between these dates you have to subtract one date from another.

For example, in 1513 Ponce de Leon explored parts of Florida, looking for the "Fountain of Youth." Then in 1527, Cabeza de Vaca began a journey of discovery and


©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

exploration, which lasted eight years. To know how many years were between Ponce de Leon's travels and those of de Vaca, you have to subtract 1513 from 1527 to get 14 years as the answer. Cabeza de Vaca spent eight years exploring what is today the American Southwest. If you wanted to know in what year he ended his amazing journey you could add eight to the year, 1527. Mr. de Vaca's journey, therefore, ended in 1535.

Professor K will take you to different places in the world and to different times so that you can learn how mathematics has been a basic tool used by mankind throughout history. In Chapter 1 you will learn the basic addition and subtraction operations on which all mathematics is built. Addition and subtraction are possible because numbers have absolute values, which allow people to calculate quantities of objects. Over the passage of years, people have been able to establish certain rules, laws, principles and properties by which to use numbers in daily situations. In this chapter you will learn about properties of addition and subtraction. A *property* is the term used to refer to a rule or principle about mathematics.




Meet ***Max Potential***, PAC curriculum's mascot. Max is an adolescent wolf who will be "hanging around" to encourage you to reach your maximum potential as you read and work through these pages.


©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

"The study of Mathematics cannot be replaced by any other activity that will train and develop man's purely logical faculties to the same level of rationality."

**—C. O. Oakley**




©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

# CHAPTER 1
# TABLE OF CONTENTS


PAGE

**SECTION 1**



1. ADDITION . . . . . .1-3
   *Identity Property, Commutative Property*



2. VALUABLE PLACES . . . . . .5-7
   *One Plus Two Digits, Two Plus Two Digits, Place Value*



3. MULTIPLE DIGITS . . . . . .9-10
   *Three Plus Two Digits, Three Plus Three Digits, Multiple Digits*



4. SUBTRACT . . . . . .11-13
   *One Minus One Digits, Two Minus One Digits, Two Minus Two Digits, Identity Property*



5. CARRY ON . . . . . .15-18
   *Carrying*

**SECTION 2**



6. TO BORROW, OR NOT . . . . . .19-22
   *Borrowing*



7. WORD SOLUTIONS . . . . . .23-27
   *Addition, Subtraction*



8. MULTIPLY . . . . . .29-33
   *Terms and Properties*

PAGE



9. MULTIPLY EVEN MORE . . . . .35-37
   *Using Larger Numbers*



10. NO REMAINS, DIVISION . . . . .39-43
    *Definition of Terms, Short Division*

**SECTION 3**



11. REMAINS, DIVISION . . . . . .45-46
    *Long Division, Terms and Steps for Solving*



12. MORE STORY PROBLEMS . . .47-50
    *Multiplication, Division, Average*



13. ENGLISH LENGTH . . . . . .51-54
    *Abbreviations, Conversions*



14. METRIC LENGTH . . . . . .55-56
    *Prefixes, Abbreviations, Conversions*



15. ENGLISH VOLUME . . . . . .57-59
    *Abbreviations, Conversions*

MATHEMATICS CHARTS . . . . . .61-63




©Copyright 2005, Paradigm Accelerated Curriculum



DUPLICATION BY WRITTEN PERMISSION ONLY.

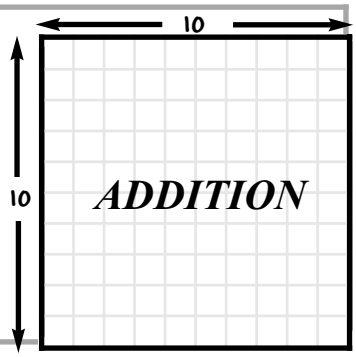
# Basic Math Skills




**Lesson 1**

*Fundamentals*

# Chapter 1




## Section 1

***Lessons 1-5***






**Professor K**

In this section you will be taught by Professor K. He will help you learn basic mathematics. His knowledge, experience and patience has helped other students just like you learn that math can be understandable and enjoyable. Basic math is a "tool" used by humans to record information, calculate strategy, discover principles and solve mysteries. Math helps mankind gain employment, accumulate wealth, invent mechanical equipment, explore space and preserve freedom. Math is basic to every vehicle that transports you, every computer you use and every weapon discharged to keep you free from tyranny.



This lesson introduces you to words, properties and principles. The first ***property*** of ***addition*** is the property of *Identity*. A number is identified when nothing (zero) is added.



**IDENTITY PROPERTY OF ADDITION**

The ***sum*** of any number and zero is the number.




***Property:*** refers to a rule or principle of mathematics
***Addition:*** the process of adding two or more numbers
***Sum:*** the answer to an addition problem
***Solution:*** the answer to any problem

**Example Set 1**

Look at the following examples of the *Identity Property of Addition*.

| | | |
|---|---|---|
| 1 + 0 = 1 | 3 + 0 = 3 | 4 + 0 = 4 |
| 6 + 0 = 6 | 14 + 0 = 14 | 134 + 0 = 134 |

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

The next property is one that deals with numbers moving to a different location or commuting to another location in the problem. The name of this property comes from a word meaning *to move*. We also have the terms "commuter train" and "commute to work" from this same root word. This property is called The *Commutative Property* because it allows numbers to move around in the problem without changing the ***solution***.



**COMMUTATIVE PROPERTY OF ADDITION**

Two numbers can be added together in either order without changing the sum.



**Example Set 2**

Examples of the *Commutative Property of Addition* are as follows:

1 + 3 = 3 + 1, because 1 + 3 = 4, and 3 + 1 = 4;
35 + 5 = 5 + 35, because 35 + 5 = 40, and 5 + 35 = 40

The answer in an addition problem is the sum.

**Example Set 3**

2 + 5 = 7 .....7 is the sum of 2 and 5

1 + 3 = 4 .....4 is the sum of 1 and 3

33 + 5 = 38 .....38 is the sum of 33 and 5

The simplest form of addition is adding one single digit to another single digit. For example:

**Table 1.1**

| | | | | |
|---|---|---|---|---|
| 1 + 0 = 1 | 6 + 1 = 7 | 1 + 5 = 6 | 7 + 2 = 9 | 2 + 1 = 3 |
| 4 + 0 = 4 | 2 + 6 = 8 | 4 + 5 = 9 | 3 + 3 = 6 | 5 + 4 = 9 |
| 1 + 3 = 4 | 7 + 0 = 7 | 1 + 8 = 9 | 9 + 0 = 9 | 2 + 4 = 6 |
| 4 + 3 = 7 | 3 + 1 = 4 | 5 + 2 = 7 | 1 + 1 = 2 | 6 + 2 = 8 |
| 1 + 6 = 7 | 8 + 0 = 8 | 2 + 2 = 4 | 4 + 1 = 5 | 2 + 7 = 9 |
| 5 + 0 = 5 | 3 + 4 = 7 | 6 + 0 = 6 | 1 + 4 = 5 | 7 + 1 = 8 |
| 2 + 0 = 2 | 3 + 6 = 9 | 2 + 5 = 7 | 4 + 4 = 8 | 3 + 2 = 5 |
| 5 + 3 = 8 | 1 + 2 = 3 | 6 + 3 = 9 | 1 + 7 = 8 | 8 + 1 = 9 |
| 2 + 3 = 5 | 4 + 2 = 6 | 3 + 0 = 3 | 5 + 1 = 6 | 3 + 5 = 8 |

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

If you look closely at **Table 1.1** on the previous page, you should be able to pick out many examples of the *Identity Property of Addition* and the *Commutative Property of Addition.*

**Example Set 4**

| Example | Property |
|---|---|
| 2 + 7 = 9 or 7 + 2 = 9 | Commutative Property |
| 1 + 8 = 9 or 8 + 1 = 9 | Commutative Property |
| 5 + 0 = 5 | Identity Property of Addition |
| 9 + 0 = 9 | Identity Property of Addition |

Apply what you have learned about addition by working the activities for this lesson.

# LIFE PRINCIPLE



"Mathematics is the language with which God wrote the universe."

—Galileo

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.




©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

# Basic Math Skills



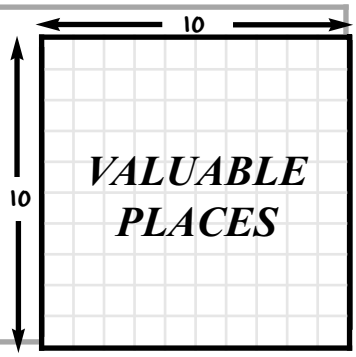

*Fundamentals*

**Lesson 2**

# Chapter 1

## Section 1

***Lessons 1-5***



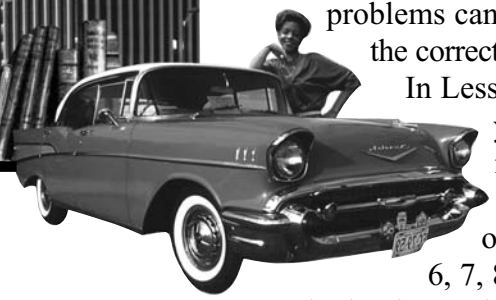






**Professor K**

Professor K teaches that a place exists for everything and everything has its place. This statement may have been mentioned to you regarding keeping items picked up in your room or keeping the house clean. Firemen need to know where their tools are stored in order to work quickly and efficiently. The same idea is true for numbers in mathematics. When we keep the digits of numbers in their proper places, math problems can be solved easily, and we get the correct answers more readily.



In Lesson 1, Professor K reminded you that the simplest numbers to add together are single ***digits***. A digit is one of the symbols, 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The size or magnitude of a number is determined by the value placed on each digit.

An easy way to remember ***place value*** is to think of numbers sitting on a bookshelf. The ***decimal point*** is like a bookend on the shelf. Numbers are like books placed on each side of the bookend. The books (numbers) have specific place values according to their positions on the shelf. **Table 2.1** shows a chart of place values to the left of the decimal point.

**Table 2.1**

| ten thousands | thousands | hundreds | tens | ones | decimal point |
|---|---|---|---|---|---|
| 10,000 | 1,000 | 100 | 10 | 1 | . |

## VOCABULARY

***Digits:*** any of the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

***Place value:*** the value given to each digit of a number; could also be referred to as the position, space, column, or simply place in the number.

***Decimal point:*** a point in all numbers directly to the right of the ones place

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

**Table 2.2** shows a chart of two numbers: *7,421* and *87,346* with each digit lined up in the proper place value column.

**Table 2.2**

| ten thousands | thousands | | hundreds | tens | ones | decimal point |
|---|---|---|---|---|---|---|
| | 7 | , | 4 | 2 | 1 | . |
| 8 | 7 | , | 3 | 4 | 6 | . |

**Example Set 1**

7,421 Notice in **Table 2.2** that *7* is in the *thousands* place, *4* is in the *hundreds*, *2* is in the *tens* and *1* is in the *ones* place.

87,346 *8* is in the *ten thousands* place, *7* is in the *thousands*, *3* is in the *hundreds*, *4* is in the *tens* and *6* is in the *ones* place.

**RULE**

When a two-digit number and a one-digit number are added together, the decimal points should be in alignment.



By aligning the decimal points, the digits of the two numbers are lined up according to the place value of each digit as in the example below.

**Example Set 2**




The *4* and *3* are in the *ones* position and are added together as single digits, giving an answer of *7*. The *6* in the *tens* position is brought down with nothing being added to the *6*. Since no number is in the *tens* position under the *6*, the place is filled with a *0* as in the equation above. Notice that when no decimal is shown, the decimal is placed to the right of the last number.

**RULE**

The decimal will always be understood to be to the right of the last digit of a number when the decimal is not shown.

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

**Example Set 3**

a) 56 + 3 = ? b) 35 + 4 = ? c) 76 + 2 = ? d) 23 + 3 = ? e) 15 + 1 = ?

| a) | b) | c) | d) | e) |
|---|---|---|---|---|
| 56. | 35. | 76. | 23. | 15. |
| + 3. | + 4. | + 2. | + 3. | + 1. |
| 59. | 39. | 78. | 26. | 16. |

The same process is true when adding two-digit numbers together as in the following example.

**Example Set 4**

a) 45 + 24 = ? b) 37 + 42 = ? c) 66 + 31 = ? d) 14 + 72 = ? e) 28 + 61 = ?

| a) | b) | c) | d) | e) |
|---|---|---|---|---|
| 45. | 37. | 66. | 14. | 28. |
| + 24. | + 42. | + 31. | + 72. | + 61. |
| 69. | 79. | 97. | 86. | 89. |

Demonstrate your understanding of place values by working the activities for this lesson.

# LIFE PRINCIPLE



*"A wise man numbers his days and is ready to give an account of his use of time."*

—Middle Eastern Proverb

©Copyright 2005, Paradigm Accelerated Curriculum

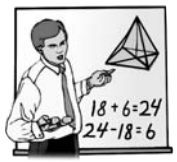
DUPLICATION BY WRITTEN PERMISSION ONLY.




©Copyright 2005, Paradigm Accelerated Curriculum

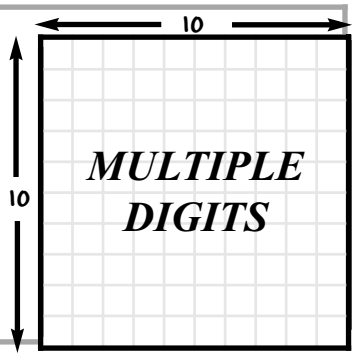
DUPLICATION BY WRITTEN PERMISSION ONLY.

# Basic Math Skills




Lesson 3

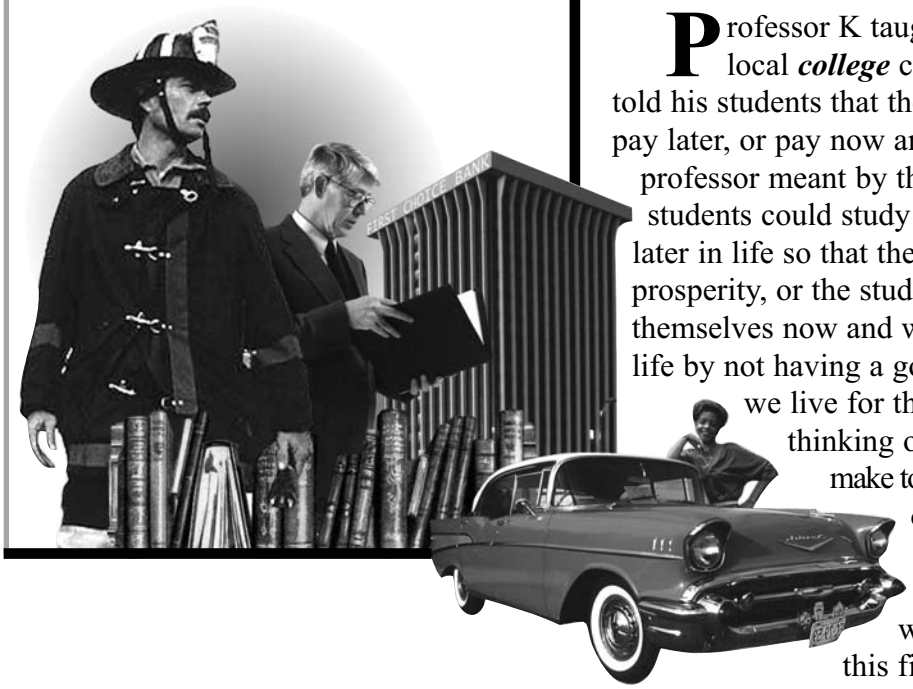
# Chapter 1




*Fundamentals*

## Section 1

***Lessons 1-5***




Professor K taught a ***physics*** class at a local ***college*** campus. The ***professor*** told his students that they could play now and pay later, or pay now and play later. What the professor meant by that statement was that the students could study now and have a good job later in life so that they could enjoy their prosperity, or the students could enjoy themselves now and work harder (pay) later in life by not having a good job. Too many times we live for the moment without thinking of our future. Choices we make today will affect us the rest of our lives.

Learning math affects you the same way. What you learn in this first chapter will help you in future lessons. Study now and future lessons will just "fall into place," one principle after another.

Professor K reminds us that addition of larger numbers is accomplished in the same manner as in the previous lesson. Look at the following examples and see how to line up numbers of two and three digits so they can be added. By aligning the decimal points, the digits in each place value are lined up so they can be added in each column. Remember, if no decimal point is shown, the point is understood to be to the right of the last digit.

### VOCABULARY

***Physics:*** a branch of science that deals with the interactions of matter and energy
***College:*** an institution for educational purposes above the high school level
***Professor:*** a teacher at a college

**Example Set 1**

hundreds, tens, ones

| a) | b) | c) | d) | e) |
|---|---|---|---|---|
| 234. | 735. | 183. | 472. | 368. |
| + 52. | + 41. | + 15. | + 23. | + 11. |
| 286. | 776. | 198. | 495. | 379. |



©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

**Example Set 1** *continued*

| f) | g) | h) | i) | j) |
|---|---|---|---|---|
| 476. | 314. | 864. | 265. | 651. |
| + 213. | + 484. | + 135. | + 721. | + 333. |
| 689. | 798. | 999. | 986. | 984. |

In the **Example Set 1** (a - j), the numbers in the *ones* place are added together, then the digits in the *tens* and *hundreds* places of each number are added. We know that when no number occupies a certain place, the space value is considered to be *0*. Therefore, *0* added to the left of a number does not change the value of that number.

**Example Set 2**

| a) | b) | c) | d) | e) |
|---|---|---|---|---|
| 5432. | 6754. | 42762. | 21864. | 64738291. |
| + 352. | + 3134. | + 7124. | + 56132. | + 241705. |
| 5784. | 9888. | 49886. | 77996. | 64979996. |

The problems in **Example Set 2** show the addition of numbers consisting of more than three digits. Notice how the place values for each number are aligned for the addition of the numbers.

Professor K encourages you to complete the activity questions to demonstrate your knowledge of adding large numbers.

# LIFE PRINCIPLE



"Work on your reputation until it is established; when it is established, it will work for you."

—Tunisian Proverb

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

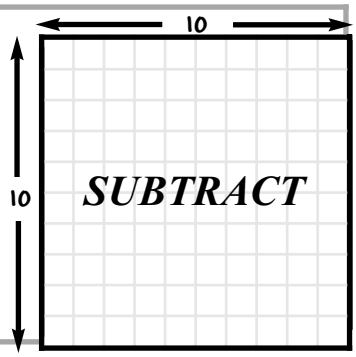
# Basic Math Skills




Lesson 4

*Fundamentals*

# Chapter 1




## Section 1

***Lessons 1-5***

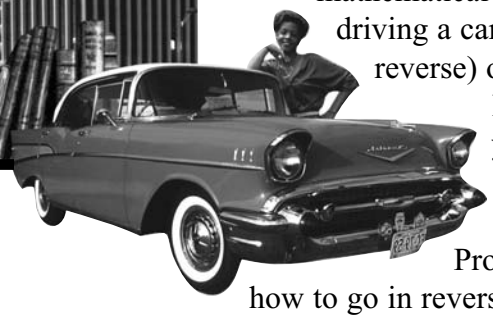





**Professor K**

Professor K and his wife enjoy automobile rides along country roads. One of the first things Professor K learned when driving a car was that a car will go forward and backward. His driving instructor made sure that Professor K put the car in drive to go forward and ***reverse*** to go backward.

Professor K reminds us that mathematical operations are similar to driving a car. The ***opposite*** (or reverse) of addition is ***subtraction***. In the previous lessons you learned how to add (drive) several types of numbers. In this lesson Professor K will teach you how to go in reverse, or subtract.



**Example Set 1**

6 - 4 = 2 ……..... *2* is the ***difference*** between *6* and *4*
3 - 2 = 1 ……..*1* is the difference between *3* and *2*
9 - 4 = 5 ……..…..*5* is the difference between *9* and *4*
7 - 3 = 4 ……..*4* is the difference between *7* and *3*

## VOCABULARY

***Reverse:*** backward
***Opposite:*** to be so different as to negate the other
***Subtraction:*** the opposite of addition
***Difference:*** the answer to a subtraction problem

Professor K reminds us that a zero is an important mathematical tool.



**IDENTITY PROPERTY OF SUBTRACTION**

If zero is subtracted from a number the result is the number.

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

**Example Set 2**

Look at the following examples of the Identity Property of Subtraction.

1 - 0 = 1  3 - 0 = 3  4 - 0 = 4  6 - 0 = 6  14 - 0 = 14  134 - 0 = 134

**Table 4.1**

| | |
|---|---|
| 1 - 0 = 1 | 7 - 0 = 7 |
| 1 - 1 = 0 | 7 - 1 = 6 |
| 2 - 0 = 2 | 7 - 2 = 5 |
| 2 - 1 = 1 | 7 - 3 = 4 |
| 2 - 2 = 0 | 7 - 4 = 3 |
| 3 - 0 = 3 | 7 - 5 = 2 |
| 3 - 1 = 2 | 7 - 6 = 1 |
| 3 - 2 = 1 | 7 - 7 = 0 |
| 3 - 3 = 0 | 8 - 0 = 8 |
| 4 - 0 = 4 | 8 - 1 = 7 |
| 4 - 1 = 3 | 8 - 2 = 6 |
| 4 - 2 = 2 | 8 - 3 = 5 |
| 4 - 3 = 1 | 8 - 4 = 4 |
| 4 - 4 = 0 | 8 - 5 = 3 |
| 5 - 0 = 5 | 8 - 6 = 2 |
| 5 - 1 = 4 | 8 - 7 = 1 |
| 5 - 2 = 3 | 8 - 8 = 0 |
| 5 - 3 = 2 | 9 - 0 = 9 |
| 5 - 4 = 1 | 9 - 1 = 8 |
| 5 - 5 = 0 | 9 - 2 = 7 |
| 6 - 0 = 6 | 9 - 3 = 6 |
| 6 - 1 = 5 | 9 - 4 = 5 |
| 6 - 2 = 4 | 9 - 5 = 4 |
| 6 - 3 = 3 | 9 - 6 = 3 |
| 6 - 4 = 2 | 9 - 7 = 2 |
| 6 - 5 = 1 | 9 - 8 = 1 |
| 6 - 6 = 0 | 9 - 9 = 0 |



**Table 4.1** shows differences when subtracting single digit numbers. Also, if you examine **Table 4.1** closely you can see examples of the Identity Property of Subtraction you learned in **Example Set 2**.

**Example Set 3** shows examples of subtraction of several (multiple) digit numbers. Notice the digits are once again lined up using the decimal point. (Remember, if not shown, the decimal point is understood to be to the right of the last digit.) This puts the digits in their proper columns according to place values. The subtraction is then performed on each column of numbers as single digits. Notice that some spaces include no number. A zero is understood to occupy each space that is not occupied by a number.

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

**Example Set 3**

a) 36 - 4 = ? b) 29 - 2 = ? c) 48 - 37 = ? d) 94 - 71 = ? e) 57 - 36 = ?

| a) | b) | c) | d) | e) |
|---|---|---|---|---|
| 36. | 29. | 48. | 94. | 57. |
| - 4. | - 2. | - 37. | - 71. | - 36. |
| 32. | 27. | 11. | 23. | 21. |

Practice your subtraction skills by working the activities for this lesson.

## LIFE PRINCIPLE



"Multiple acts soon define your character."

—American Proverb




©Copyright 2005, Paradigm Accelerated Curriculum

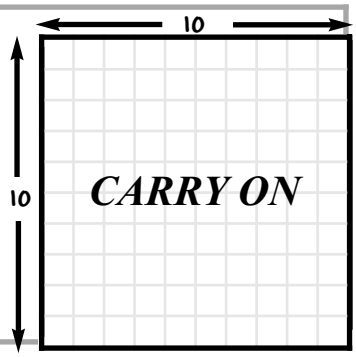
DUPLICATION BY WRITTEN PERMISSION ONLY.




Lesson 5

*Fundamentals*

# Chapter 1




## Section 1

***Lessons 1-5***






**Professor K**

Professor K, like most people, uses banks for two basic purposes: 1) to "***carry***" extra money there for safekeeping, or to earn ***interest***, and 2) to "borrow" money for large purchases such as cars or houses. When adding, you sometimes will have more numbers than needed, so you "carry." Sometimes when subtracting, you need more numbers, so you "borrow." In this lesson, Professor K helps you review when and how to carry in addition problems.



When two digits are added together, and the ***sum*** is greater than 9, the digit in the tens position of the sum is carried over into the next higher place value, or into the column to the left.

Notice how Professor K's examples below demonstrate this addition principle of carrying.

## VOCABULARY

***Carry:*** action when numbers are added and the sum in a column is greater than ten, one or more groups of ten are moved to the column to the left.
***Interest:*** money earned on money loaned to another person
***Sum:*** the answer in an addition problem

**Example Set 1**

a) 6 + 4 = 10. *10* is one group of *tens*, plus zero *ones*.
b) 4 + 8 = 12. *12* is one group of *tens*, plus two *ones*.
c) 9 + 8 = 17. *17* is one group of *tens*, plus seven *ones*.
d) 7 + 7 = 14. *14* is one group of *tens*, plus four *ones*.
e) 9 + 3 = 12. *12* is one group of *tens*, plus two *ones*.

| a) | b) | c) | d) | e) |
|---|---|---|---|---|
| $^{1}6.$ | $^{1}4.$ | $^{1}9.$ | $^{1}7.$ | $^{1}3.$ |
| $+\ 4.$ | $+\ 8.$ | $+\ 8.$ | $+\ 7.$ | $+\ 9.$ |
| **1**0. | **1**2. | **1**7. | **1**4. | **1**2. |

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

In the following examples, the one is carried to the tens column where it is added to the other numbers in the tens column, unless no number is there (in the tens place). In that case, the space is considered to have a zero in that position, as shown in **Example Set 1**.

**Example Set 2**

a) 15 + 6 = 21. *21* is two groups of *tens,* plus *1* group of *ones*.
b) 48 + 5 = 53. *53* is five groups of *tens*, plus *3* groups of *ones*.
c) 27 + 54 = 81. *81* is eight groups of *tens*, plus *1* group of *ones*.
d) 73 + 18 = 91. *91* is nine groups of *tens*, plus *1* group of *ones*.
e) 56 + 26 = 82. *82* is eight groups of *tens*, plus *2* groups of *ones*.

| a) | b) | c) | d) | e) |
|---|---|---|---|---|
| $\overset{1}{15}$. | $\overset{1}{48}$. | $\overset{1}{27}$. | $\overset{1}{73}$. | $\overset{1}{56}$. |
| + 6. | + 5. | + 54. | + 18. | + 26. |
| 21. | 53. | 81. | 91. | 82. |

Many times in life, you will want to add more than two numbers together. This means that often a number that is greater than 1 is carried over to the *tens* place. In order to add more than two numbers together, another mathematical property must be stated.

Professor K now introduces you to another tool you can use in mathematics. The *Associative Property* is easily remembered when you realize that the order in which you introduce two friends makes no difference. Friends are sometimes called associates. One can be introduced before the other or vice versa. The order of introduction does not matter.



**ASSOCIATIVE PROPERTY OF ADDITION**

When three or more numbers are added, the order in which they are added does not matter. (Stated another way: the answer remains the same regardless of the order of addition of the numbers.)

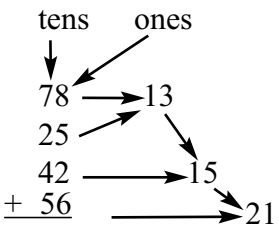


**Example Set 3**

(4 + 5) + 9 = 4 + (5 + 9)  1 + (3 + 7) = (1 + 3) + 7

Professor K puts several principles and properties together to help you learn how to add multiple numbers in the *tens* and *ones* places.

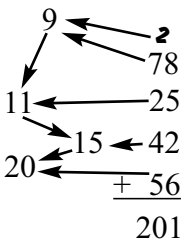
**Example Set 4**




Adding the *ones* column gives: *8 + 5 = 13*; then *13 + 2 = 15*; then *15 + 6 = 21*.

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.




The *1* in *21* is placed in the *ones* column. The *2* in *21* is carried to the *tens* column. Now adding the *tens* column gives: *2 + 7 = 9*; then *9 + 2 = 11*; and *11 + 4 = 15*; and finally, *15 + 5 = 20*. The *0* is placed in the *tens* space. The *2* in the sum of the *tens* column is carried to the *hundreds* column; and since there are no other numbers in that column, the *2* is added to *0* and written in the answer.

Addition with carrying can also be used in addition of larger numbers. Professor K provides **Example Set 5** to guide you through the steps necessary to complete problems which require carrying numbers.

**Example Set 5**




```
 3683.
+ 794.
```

In this problem, a 3-digit number, *794*, is added to a 4-digit number, *3,683*. The sum of the digits in the *ones* column is *7*. Since you have nothing to carry, *7* is written in the *ones* place of the answer. (See Step 2.)




```
   1
 3683.
+ 794.
  77.
```

The sum of the digits in the *tens* column is *17*. In this case, the *7* is placed in the *tens* place of the answer, and the *1* is carried to the *hundreds* column to be added to the digits in that column. (See Step 2.)




```
  1 1
 3683.
+ 794.
 4477.
```

The *1* that was carried to the *hundreds* column is now added to the *6* and *7*, which are already in the *hundreds* column, to give a sum of *14*. The *4* is placed in the *hundreds* place in the answer and the *1* is carried to the *thousands* column and added to the *3* already there to get a sum of *4*, which is placed in the *thousands* place in the answer. This gives a sum of *4,477* for the problem. (See Step 3.)



©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

Professor K provides another example just to make sure you understand carrying from one place to another.

**Example Set 6**

**Step 1**

```
  43682.
+ 6774.
```

In this problem, a 4-digit number is added to a 5-digit number.

**Step 2**

```
   1
  43682.
+ 6774.
  56.
```

The sum of the digits in the *ones* column is *6*, therefore you have nothing to carry, and *6* is written in the ones place of the answer. (See Step 2.) The sum of the digits in the *tens* column is *15*. In this case the *5* is placed in the *tens* column of the answer, and the *1* is carried to the *hundreds* column to be added to the digits in that column. (See Step 2.)

**Step 3**

```
  111
  43682.
+ 6774.
  0456.
```

The *1* that was carried to the *hundreds* column is now added to the *6* and *7*, which are already in the *hundreds* column to give a sum of *14*. The *4* is placed in the *hundreds* place in the answer and the *1* is carried to the *thousands* column and added to the *3* and the *6* already there to get a sum of *10*. The *0* is placed in the *thousands* place in the answer and the *1* is carried to the *ten thousands* place. (See Step 3.)

**Step 4**

```
  1
  43682.
+ 6774.
  50456.
```

The *1* carried in the previous step into the *ten thousands* column is added to the *4* already there to obtain a sum of *5*. This gives a sum of *50,456.* for the problem. (See Step 4.)

---

Demonstrate your understanding of addition by working the activities for this lesson. Professor K reminds you to review the vocabulary, properties and principles carefully before you request the section quiz. Remember, if you score 85% or higher on the three section quizzes in this chapter, you may be exempt from the Chapter Test following Lesson 15.

# LIFE PRINCIPLE



"The sum of all your choices is your character."

—Nathan Johnson

©Copyright 2005, Paradigm Accelerated Curriculum

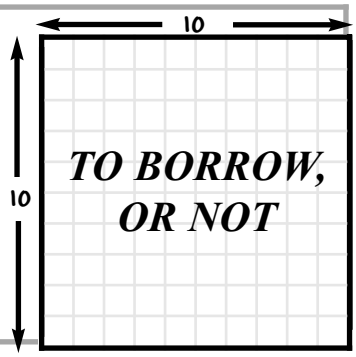
DUPLICATION BY WRITTEN PERMISSION ONLY.




Lesson 6

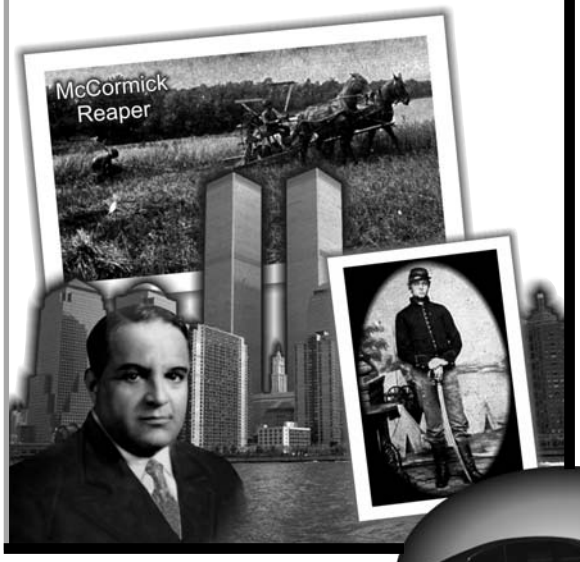
*Fundamentals*

# Chapter 1




## Section 2

***Lessons 6-10***






In Lesson 5, Professor K reviewed the concept of carrying to work addition problems. Remember how Professor K "carried" extra money to the bank is similar to carrying the value of ten into the next place value. Sometimes Professor K does not have enough cash to pay his expenses as he takes you on the journey through the land of mathematics. He must go to the bank and borrow from his ***savings account***. Sometimes when subtracting, you need more numbers so you **"*borrow*"** from the "number bank" of larger place value, which is the column to the left. In this lesson Professor K will review with you when and how to borrow in subtraction problems.

In the following example, Professor K shows you that subtracting smaller digits from larger digits is made simple by keeping the ***respective differences*** in the proper place value columns.

**Example Set 1**

| a) | b) | c) | d) | e) |
|---|---|---|---|---|
| 567. | 865. | 143. | 356. | 368. |
| - 52. | - 32. | - 22. | - 51. | - 11. |
| 515. | 833. | 121. | 305. | 357. |

In **Example Set 1** a), in the *ones* column, *2* is subtracted from *7*. In the *tens* column, *5* is subtracted from *6*. The difference of each subtraction is placed in the proper column of the answer, *5* in the *ones* column, and *1* in the *tens* column. Since no digit is in the *hundreds* place of the second number, the space is assumed to have a *0*, and the difference between 5 and 0 (which is *5*) is placed in the hundreds place of the answer .

Sometimes when subtracting, you need to borrow, or do the opposite of carrying (as you do in addition). In the

## VOCABULARY

***Savings account:*** extra money a person sets aside in a bank, to be withdrawn for emergencies or purchases

***Borrow:*** to get temporary use of; get a set of tens from the column to the left in a subtraction problem

***Respective:*** separate items related in some way to one another

***Differences:*** answers to subtraction problems

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

following math principle, Professor K explains how to go to the "number bank" in the column to the left to borrow enough numbers to accomplish subtraction.

**MATH PRINCIPLE**



To borrow means to decrease the place value of the column to the left by a value of one and place it with the next lower place value.



Professor K explains this principle more fully in **Example Set 2**.

**Example Set 2**

**Step 1**

$$\begin{array}{r} 536. \\ -\ 257. \\ \hline \end{array}$$

This problem is to subtract *257* from *536*.

**Step 2**

$$\begin{array}{r} {\scriptstyle 2\ 16} \\ 5\not{3}\not{6}. \\ -\ 257. \\ \hline 9. \end{array}$$

In this problem, *7* cannot be subtracted from *6*. Both of which are in the *ones* place; therefore, a group of ten digits must be borrowed from the *tens* place next door. The group of tens is added to the *6* to make *16*, a number from which *7* can be subtracted. The *3* in the *tens* place becomes a *2* because we borrowed one group of ten digits. (See Step 2.) Now, *7* from *16* gives a *9* in the *ones* place of the answer. By looking at the *tens* place, you see that *5* cannot be subtracted from *2*. This means that the borrowing process must be repeated in the *hundreds* place. (See Step 2.)

**Step 3**

$$\begin{array}{r} {\scriptstyle 4\ 2\ 16} \\ \not{5}\not{3}\not{6}. \\ -\ 257. \\ \hline 279. \end{array}$$

This time the group of ten digits is borrowed from the *hundreds* place, changing the *5* to a *4*. The *2* is increased to a *12* when the group of ten digits from the *hundreds* place is added to the *2*. (See Step 3.) The *5* can now be subtracted from *12*, giving *7*. Then *4 - 2 = 2*, giving *279* as the final difference between *536* and *257*.

Professor K wants you to know that you can check your own subtraction problems to make sure they are correct before handing in your work to a teacher.

**Example Set 3**

a)

$$\begin{array}{r} 567 \\ -\ 52 \\ \hline 515 \end{array} \quad \begin{array}{l} \leftarrow \\ \rightarrow + \\ \leftarrow = \end{array}$$

a) To check this problem, add the number you subtracted to the answer in each column. If your answer to the original problem was correct, the result of this check work will be the top number of the original problem. Work upwards from right to left: 5 + 2 = 7, 1 + 5 = 6, 5 + 0 = 5. The original answer (515) was correct because 515 + 52 = 567, the top number of the original problem.

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

b)
$$\begin{array}{r} 536 \\ -\ 257 \\ \hline 279 \end{array}$$

b) If you had to borrow to work the original problem, you will have to carry in order to check your work (because now you have to add, which is the opposite of subtraction). Except for carrying, the checking process is the same as Step 1. Work upwards from right to left: 9 + 7 = 16 (write the 6, carry the 1); 7 + 5 = 12 plus the 1 carried = 13 (write the 3, carry the 1); 2 + 2 = 4 plus the 1 carried = 5. The original answer was correct because 279 + 257 = 536, the top number of the original problem.

---

Subtraction with borrowing is used in subtraction of larger numbers. Professor K uses **Example Sets 4 and 5** to guide you through the steps necessary to complete such problems.

**Example Set 4**

**Step 1**

$$\begin{array}{r} 3683. \\ -\ 794. \\ \hline \end{array}$$

In this problem, a 3-digit number is subtracted from a 4-digit number.

**Step 2**

$$\begin{array}{r} {\scriptstyle 7\,13} \\ 36\not{8}3. \\ -\ 794. \\ \hline 9. \end{array}$$

The difference of the digits in the *ones* column cannot be obtained since *4* cannot be subtracted from *3*. A group of ten digits must be borrowed from the *tens* column to the left. Therefore, the *8* becomes *7* and the *3* in the *ones* column becomes *13*. (See Step 2.) Performing the subtraction, *13 - 4*, yields a difference of *9*, which is placed in the answer row. (See Step 2.)

**Step 3**

$$\begin{array}{r} {\scriptstyle 5\,17} \\ 3\not{6}\not{8}3. \\ -\ 794. \\ \hline 89. \end{array}$$

The subtraction in the *tens* column of *7 - 9* also cannot be accomplished until a group of ten digits is borrowed from the *6* in the *hundreds* column, leaving a *5*. The group of ten digits from the *hundreds* column is combined with the *7* in the *tens* column to obtain a result of *17* in the *tens* column. (See Step 3.) Performing the subtraction of the numbers in the *tens* column yields a difference of *8* which is placed in the *tens* position of the answer. (See Step 3.)



©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

**Step 4**

$$\begin{array}{r} {\scriptstyle 2\,15} \\ 3\not{6}83. \\ -\ 794. \\ \hline 2889. \end{array}$$

Look at the *hundreds* column; *5 - 7* cannot be accomplished until a group of ten digits is borrowed from the *3* in the *thousands* column, leaving a *2*. The group of *ten* digits from the *thousands* column is combined with the *5* to make *15*. (See Step 4.) The subtraction is performed in the *hundreds* column and the difference of *8* is placed in the *hundreds* position of the answer. Nothing is subtracted from the *2* in the *thousands* column, so the 2 is placed in the *thousands* position of the answer, *2,889*. (See Step 4.)

---

**Example Set 5**

**Step 1**

$$\begin{array}{r} 8538. \\ -\ 2647. \\ \hline 1. \end{array}$$

In this subtraction problem the difference of the digits, *8 - 7*, in the *ones* column is *1*. No borrowing is necessary at this time. The *1* is placed in the *ones* place of the answer. (See Step 1.)

**Step 2**

$$\begin{array}{r} {\scriptstyle 4\,13} \\ 8\not{5}\not{3}8. \\ -\ 2647. \\ \hline 91. \end{array}$$

In the *tens* column, *4* cannot be subtracted from *3*, so a group of ten digits must be borrowed from the *5*, leaving a *4* in the *hundreds* place. Notice that *13 - 4 = 9*, and *9* is placed in the *tens* place of the answer. (See Step 2.)

**Step 3**

$$\begin{array}{r} {\scriptstyle 7\,14\ 4\,13} \\ 8\not{5}\not{3}8. \\ -\ 2647. \\ \hline 5891. \end{array}$$

Notice the *hundreds* column; *6* cannot be subtracted from *4*, therefore, a group of ten digits must be borrowed from the *8* in the *hundreds* place, leaving a *7* in that place. The *4* in the *hundreds* place now becomes *14*. In the *hundreds* column, *14 - 6 = 8*, and *8* is placed in the *hundreds* place in the answer. Notice the *thousands* column; *7 - 2 = 5*, and *5* is placed in the *thousands* place in the answer, giving a final difference of *5,891*. (See Step 3.)

---

Apply what you have learned about subtraction by working the activities for this lesson.

# LIFE PRINCIPLE



"The man who borrows is servant to the man who lends."

—Hebrew Proverb

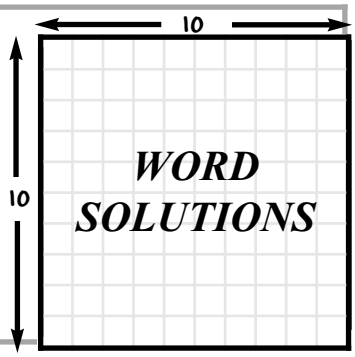
# Basic Math Skills




Lesson 7

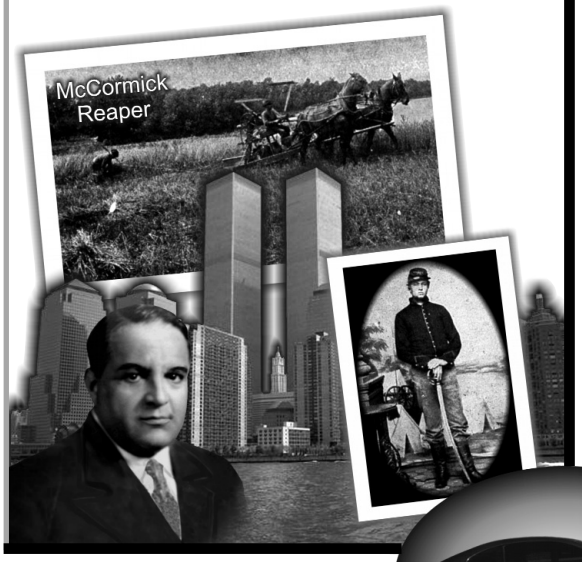
*Word Problems*

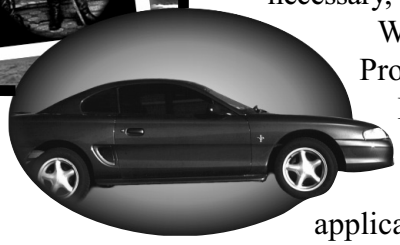
# Chapter 1




## Section 2

***Lessons 6-10***



Professor K enjoys reading and studying history. When looking at the history of the world, Professor K noticed that throughout time men and women of all walks of life and status have faced problems and emergencies. They had to learn to solve problems quickly. Problem solving was especially important to men and women leaders in cities, states and nations. During times of great hardship such as the ***Civil War***, the ***Great Depression*** or 9-11 Terrorist Attacks, leaders had to make tough, yet necessary, decisions.

While studying the Civil War, Professor K saw that Abraham Lincoln's problems were multiplied many times over as the nation he led became involved in the application of principles that divided the United States. However, President Lincoln looked at the problems at hand and solved them with perseverance and fortitude.

Another leader Professor K learned about in his studies of history was Fiorello LaGuardia, who was the mayor of New York City during the Great Depression. The mayor faced problems associated with people who were out of work and without food. Mayor LaGuardia's problems were multiplied many times over by the economic hardships associated with the depressed economy. However, instead of dividing his city, he brought it together with love and understanding.

Another New York mayor, Mario Juliani, faced serious problems on September 11, 2001 when terrorists destroyed the twin towers in New York City. Mayor Juliani faced the terrible conditions and drew the distraught city together to rescue victims and persevered through the painful loss of lives.

## VOCABULARY

***Civil War:*** the war between the North and South (Union and Confederacy) 1860 - 1865
***Great Depression:*** a period of history between 1929 - 1939
***Benefit:*** something that is useful or profitable
***Mathematical model:*** a statement using formulas, operations, or a series of operations
***Key:*** something that is important to the answer

©Copyright 2005, Paradigm Accelerated Curriculum

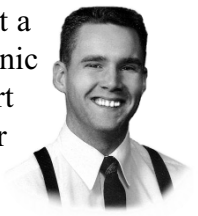
DUPLICATION BY WRITTEN PERMISSION ONLY.

These three men (through their writings and speeches) claimed to have relied on their belief in God and personal determination to accomplish the tasks they were required to perform. Professor K reminds you that math is a series of problems to be solved. You, too, can solve math and personal problems associated with life if you develop good character and learn basic math skills.

In the next few lessons Professor K will help you learn the basic method for solving word problems. Also, you will review the basics of multiplication and division.

## ADDITION AND SUBTRACTION: WORD PROBLEMS

Professor K believes that mathematics can help you cope with life. Life is a word problem. Each day you face problems that have to be solved. Not all problems you face in life involve math skills, but learning how to work through math problems one step at a time can ***benefit*** you in working through problems in life. An automobile mechanic uses words to tell you what is wrong with your car and to relate the financial part of the problem to you. He tells you that the parts cost a certain amount, the labor is priced per hour and the mechanic will need a certain number of hours to finish the job. You then must work the word problem to determine the cost of repairing your car.



The problems Professor K discusses in this lesson will include math story (word) problems related to real world situations, thereby demonstrating how math relates to your life. Word problems become more readily workable when you follow Professor K's simple steps. If you will become familiar with these steps and follow them, then word problems will be as easy for you as they are for Professor K.

**Steps to solving a word problem.**

***Step 1*** Read the problem through completely, not worrying about any numbers that are involved. This step gives you an idea of what facts are presented in the problem and for what the problem is asking. Reread until the problem is understood. (Remember, President Lincoln and Mayors LaGuardia and Juliani had to survey the areas of disaster before developing solutions.)

***Step 2*** After rereading the problem, take note of the numbers involved in the problem, and even jot down the numbers in the margin of your activity book if necessary for future use. (The President and two New York City mayors made notes of important events, casualties and losses.

***Step 3*** By looking at the wording of the problem (refer to the list of "***key***" words on the following page) determine what mathematical operation(s) or model will be required to find the solution to the problem (explained in **Example Sets 1** and **2**). Study the numbers used in the problem and put them in the proper places of the ***mathematical model*** according to the way the problem is worded (explained in **Example Sets 1** and **2**).

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

**Step 4** After completing Step 3, work the problem and obtain the solution (answer).

**Step 5** Look at the answer to see if it is a reasonable solution to the problem. Ask yourself whether the answer looks right or makes sense in relation to the numbers in the problem.

As explained in Step 3 above, Professor K looks for "key" words in a story problem that will tell him what operation (addition or subtraction) is involved. Some of these "key" words are listed below for addition and subtraction story problems.

**"Key" words used in story problems and what they mean.**

| | |
|---|---|
| all together, plus | addition problem (+) |
| sum, increase, total | addition problem (+) |
| how much more, take away | subtraction problem (-) |
| difference, less, decrease | subtraction problem (-) |

Follow the steps in **Example Sets 1** and **2** as Professor K guides you through the word problems contained in the examples.

**Example Set 1**

If Mayor Juliani called *1,200* firemen and *600* policemen to the Twin Towers, how many emergency personnel all together responded to the disaster?
Following the instructions in **Steps to solving a word problem**, the problem is worked by the following procedure:

**Step 1** Read the problem to see that two groups, *policemen* and *firemen*, responded. The problem is to find out the amount of people involved in emergency procedures.

**Step 2** Rereading the problem, you notice that one group has *1,200* people and the other has *600*.

**Step 3** The wording of the problem ("all together") shows that the operation needed to solve the problem is *addition*.

**Step 4** Add

$$\begin{array}{rl} 1200. & \text{amount of firemen} \\ +\ 600. & \text{amount of policemen} \\ \hline 1800. & \text{amount of total emergency personnel} \end{array}$$



**Step 5** Check to see whether the answer seems reasonable.

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

**Example Set 2**

Robert and James were volunteers who distributed emergency water containers to rescue workers. Each boy was given a box containing *50* bottles. Robert gave away *35* bottles and James distributed *41* bottles.

a) How many bottles all together were given to Robert and James?
b) How many bottles did Robert and James distribute all together?
c) How many more bottles did James distribute than did Robert?

Following the steps in **Steps to solving a word problem**, the problem is worked by the following procedure:

***Step 1*** After reading the problem, not worrying about the numbers, you notice that two boys were distributing bottles. Each boy was given a box of bottles, and both boys gave away some of the bottles. The problem then asks three questions: How many total bottles were given to the boys? How many total bottles were distributed? and, What is the difference between the quantities James distributed compared to Robert?

***Step 2*** Reread the problem to consider the numbers. Jot down the following numbers: *2* boys, *1* box of bottles = *50* bottles, Robert distributed *35* bottles, and James gave away *41* bottles.

***Step 3*** Look at the questions:

a) The wording of the question, *all together*, translates into an addition problem

50. bottles given to Robert
+ 50. bottles given to James
100. bottles is total for both boys

b) In this question, *all together*, also translates into an addition problem, by looking at how many bottles each boy distributed, and then adding them together

35. bottles given away by Robert
+ 41. bottles given away by James
76. total bottles given away by both boys

c) The last question asks *how many more* bottles one boy gave away than were given away by the other boy. Therefore, this will be a subtraction problem since the problem is asking for the *difference* between the numbers of bottles distributed by the two boys.



41. bottles distributed by James
- 35. bottles distributed by Robert
6. James gave away 6 more bottles than Robert gave away.

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

***Step 5*** Now check your answers to see if they seem reasonable.

Now complete your activities and check your answers. Tell yourself, "I am learning to solve word problems."

# LIFE PRINCIPLE



**"Heroes are average people who respond immediately to the needs of people in crisis."**

—Anonymous

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.




©Copyright 2005, Paradigm Accelerated Curriculum

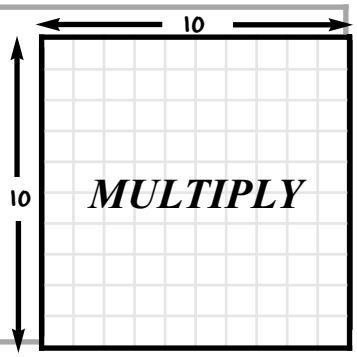
DUPLICATION BY WRITTEN PERMISSION ONLY.




Lesson 8

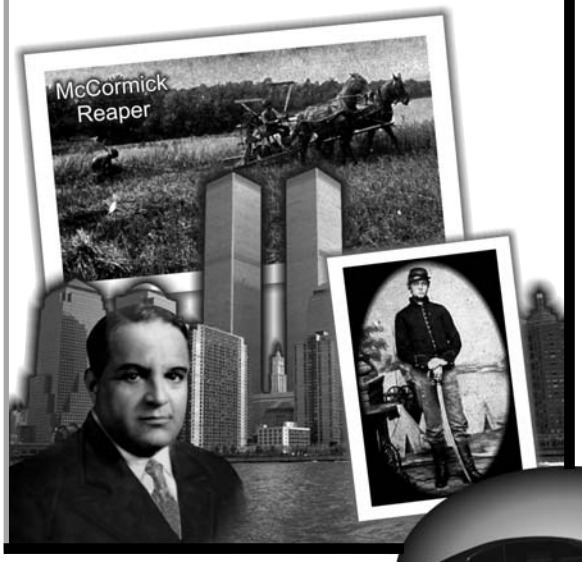
*Fundamentals*

# Chapter 1




## Section 2

***Lessons 6-10***








**Professor K**

You probably remember reading about a period of world history in the 1800s known as the ***Industrial Revolution***. This was a time when inventors were working hard to invent machines of every possible type to make work easier or quicker. Cyrus McCormick was the man who invented a machine called a reaper to make harvesting crops quicker, easier and with fewer laborers. The reaper was a shortcut used by farmers to harvest crops.

Mathematics allows you to take many shortcuts. Multiplication is a shortcut to addition. Professor K likes to take shortcuts as often as he can so he ***memorized*** the multiplication tables. Study this lesson and the next as Professor K reviews the multiplication tables with you for the numbers *1 - 12.* Remember, if you haven't memorized these tables yet, Professor K advises you to do so to make solving math problems easier and quicker.




***Industrial Revolution:*** the rapid development of industry by use of machines in the 19th century
***Memorized:*** repeated until known and remember instantly
***Product:*** the answer in a multiplication problem
***Factor:*** numbers that are multiplied together
***Various:*** different kinds

**Example Set 1**

Example: 3 × 5 = 15; 15 is the ***product*** of 3 & 5
Example: 4 × 6 = 24; 24 is the product of 4 & 6

**Table 8.1**

| ADDITION | | MULTIPLICATION |
|---|---|---|
| 2 | = 2 | 1 × 2 = 2 |
| 2 + 2 | = 4 | 2 × 2 = 4 |
| 2 + 2 + 2 | = 6 | 3 × 2 = 6 |
| 2 + 2 + 2 + 2 | = 8 | 4 × 2 = 8 |
| 2 + 2 + 2 + 2 + 2 | = 10 | 5 × 2 = 10 |
| 2 + 2 + 2 + 2 + 2 + 2 | = 12 | 6 × 2 = 12 |

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

You can see by comparing the *Addition* column and the *Multiplication* column, that when *2* is added twice, the answer is the same as multiplying *2* by *2*. The same is true for *2* added three times, four times, five times, and six times, or *2* multiplied by *3*, *4*, *5*, and *6*.

Even though you can add a number as many times as needed, a shortcut to addition is simply to memorize multiplication tables. Memorizing the multiplication tables is the best way to get the answer quickly.

Professor K has provided multiplication tables for you to memorize because he knows that students who memorize these multiplication tables will jump ahead of others who put off learning these shortcuts to mathematical problems.

**Table 8.2** is a multiplication table for numbers *1* - *6*. The multiplication table for numbers *7* - *12* will be presented in Lesson 9. Numbers multiplied together are called ***factors*** of the product. The numbers to the left of an equals sign are factors of the product to the right of the equal sign.

**Table 8.2**

factors product

Example: $2 \times 3 = 6$, *2* and *3* are factors of *6* (the product)
Example: $4 \times 5 = 20$, *4* and *5* are factors of *20* (the product)



| | | |
|---|---|---|
| $1 \times 1 = 1$ | $2 \times 1 = 2$ | $3 \times 1 = 3$ |
| $1 \times 2 = 2$ | $2 \times 2 = 4$ | $3 \times 2 = 6$ |
| $1 \times 3 = 3$ | $2 \times 3 = 6$ | $3 \times 3 = 9$ |
| $1 \times 4 = 4$ | $2 \times 4 = 8$ | $3 \times 4 = 12$ |
| $1 \times 5 = 5$ | $2 \times 5 = 10$ | $3 \times 5 = 15$ |
| $1 \times 6 = 6$ | $2 \times 6 = 12$ | $3 \times 6 = 18$ |
| $1 \times 7 = 7$ | $2 \times 7 = 14$ | $3 \times 7 = 21$ |
| $1 \times 8 = 8$ | $2 \times 8 = 16$ | $3 \times 8 = 24$ |
| $1 \times 9 = 9$ | $2 \times 9 = 18$ | $3 \times 9 = 27$ |
| $1 \times 10 = 10$ | $2 \times 10 = 20$ | $3 \times 10 = 30$ |
| $1 \times 11 = 11$ | $2 \times 11 = 22$ | $3 \times 11 = 33$ |
| $1 \times 12 = 12$ | $2 \times 12 = 24$ | $3 \times 12 = 36$ |

| | | |
|---|---|---|
| $4 \times 1 = 4$ | $5 \times 1 = 5$ | $6 \times 1 = 6$ |
| $4 \times 2 = 8$ | $5 \times 2 = 10$ | $6 \times 2 = 12$ |
| $4 \times 3 = 12$ | $5 \times 3 = 15$ | $6 \times 3 = 18$ |
| $4 \times 4 = 16$ | $5 \times 4 = 20$ | $6 \times 4 = 24$ |
| $4 \times 5 = 20$ | $5 \times 5 = 25$ | $6 \times 5 = 30$ |
| $4 \times 6 = 24$ | $5 \times 6 = 30$ | $6 \times 6 = 36$ |
| $4 \times 7 = 28$ | $5 \times 7 = 35$ | $6 \times 7 = 42$ |
| $4 \times 8 = 32$ | $5 \times 8 = 40$ | $6 \times 8 = 48$ |
| $4 \times 9 = 36$ | $5 \times 9 = 45$ | $6 \times 9 = 54$ |
| $4 \times 10 = 40$ | $5 \times 10 = 50$ | $6 \times 10 = 60$ |
| $4 \times 11 = 44$ | $5 \times 11 = 55$ | $6 \times 11 = 66$ |
| $4 \times 12 = 48$ | $5 \times 12 = 60$ | $6 \times 12 = 72$ |

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

Memorizing the multiplication facts in **Table 8.2** will make your work less complicated when faced with larger multiplication problems. You can work larger multiplication problems by first working several smaller problems, which are contained in the larger problem.

Along with memorizing your multiplication tables, learning certain rules and properties help make math problems easier to solve. The *Multiplication Property of One and Zero* are two such properties. Look at these properties closely before moving on to the examples.



**MULTIPLICATION PROPERTY OF ZERO**
The product of zero and any number is zero.



**Example Set 2**

Example: $4 \times 0 = 0$
Example: $6 \times 0 = 0$
This fact is known as the *Multiplication Property of Zero.*



**IDENTITY PROPERTY OF MULTIPLICATION**
Any number multiplied by one remains the same number.

**Example Set 3**

$4 \times 1 = 4$
$34 \times 1 = 34$
$57 \times 1 = 57$
$542 \times 1 = 542$
$5983 \times 1 = 5983$

The following example shows you how to solve a multiplication problem involving a two-digit number multiplied by a one-digit number.

**Example Set 4**



***Step 1***

$$\begin{array}{r} 56. \\ \times\ 6. \\ \hline \end{array}$$

In this problem, *56* is multiplied by *6*. Adding *56* to itself *6* times would produce the answer; however, multiplication provides the answer much faster.



***Step 2***

$$\begin{array}{r} {\scriptstyle 3} \\ 56. \\ \times\ 6. \\ \hline 6. \end{array}$$

To begin to work a multiplication problem such as this, the numbers in the *ones* column are multiplied together first. In this example, $6 \times 6 = 36$. The *6* is placed in the *ones* place of the answer, and the *3* is carried to the *tens* column. (See Step 2.)

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

**Step 3**

```
  3
 56.
×  6.
336.
```

The next step to solve this multiplication problem is to multiply *6* times *5*. (*6* × *5* = 30). The answer to this smaller multiplication problem, *30*, is now added to the carried number (30 + 3 = 33). Since no number exists in the *hundreds* place, the *33* is placed in the answer as shown. This makes the product of 56 × 6 = 336. (See Step 3.)

---

You will learn in **Example Set 5** how to multiply a three-digit number by a two-digit number using numbers concerning the 9-11 terrorists' attacks on the Twin Towers. If *256* rescue workers worked *32* straight hours without rest, how many total hours did they work before resting?

**Example Set 5**

**Step 1**

```
 256.
× 32.
```

This problem requires multiplying *256* by *32*. Compared to the previous example, a few more steps are required to solve this problem.

**Step 2**

```
  11
 256.
× 32.
 12.
```

Look first at the *ones* column; work only with the *2* in *32* and the *6* in *256*. Notice that *2* × *6* = *12*. The *2* is placed in the *ones* place of the first answer row and the *1* is carried to the *tens* column to be added to the product of the digits in the *tens* column. The next step is 2 × 5 = *10*, and *10* + *1* = *11*. The *1* in the *ones* place of this answer is placed in the *tens* place of the first answer row and the other *1* is carried to the *hundreds* column as shown. (See Step 2.)

**Step 3**

```
  11
 256
× 32
 512
```

The final step for this part of the problem is *2* × *2* = *4*, and *4* + *1* = *5*. The *5* is placed in the *hundreds* place of the first answer row, so *512* is the first answer needed to solve the problem. (See Step 3.)

**Step 4**

```
  1
 256.
× 32.
 512.
 80.
```

A zero (*0*) in the second answer row marks off the *ones* place. Because we are multiplying by a digit in the *tens* place, the next product lines up under the *3*, because *3* is in the *tens* place. Now, working across the top line of numbers from right to left as we did in Steps 2 and 3 you notice that, *3* × *6* = *18*. The *8* is placed in the *tens* place of the second answer row, and the *1* is carried above the *5* in *256*. (See Step 4.)

**Step 5**

```
  11
 256.
× 32.
 512.
 680.
```

In the next step, *3* × *5* = *15*, and *15* + *1* = *16*. The *6* is placed beside the *8* in the second answer row and the *1* is carried above the *2* in *256*. (See Step 5.)

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

**Step 6**

$$\begin{array}{r} {\scriptstyle 1\,1} \\ 256. \\ \times\ 32. \\ \hline 512. \\ +\ 7680. \\ \hline 8192. \end{array}$$

Notice, $3 \times 2 = 6$, and $6 + 1 = 7$. The *7* is placed in front of the *6* in the second answer row. (See Step 6.) Now add the first answer row to the second answer row to obtain the final product, *8192,* for the problem. (See Step 6.)

---

Professor K provides the following principle for multiplying numbers containing ***various*** numbers of digits. **To multiply factors consisting of a larger number of digits, the process is the same as with factors of a smaller number of digits.**

Professor K wants you to go to the activities for this lesson and demonstrate your skill at working multiplication problems. Apply the principle stated above and take small steps to reach your destination.

# LIFE PRINCIPLE



***Heroes are known by their actions during a crisis.***

—Anonymous

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.




©Copyright 2005, Paradigm Accelerated Curriculum

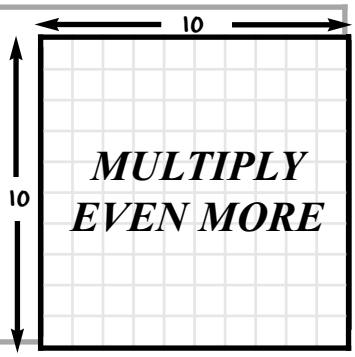
DUPLICATION BY WRITTEN PERMISSION ONLY.




Lesson 9

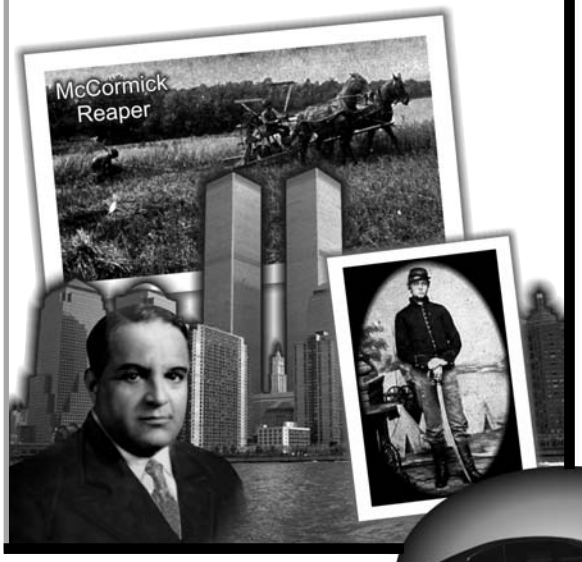
*Fundamentals*

# Chapter 1




## Section 2

***Lessons 6-10***








**Professor K**

The Industrial Revolution is said to have ended in the early 1900s. So many new ***inventions*** had been produced that the director of the ***U.S. Patent Office*** suggested that the office could be closed because nothing remained to be invented. However, inventors continued to place numerous inventions on the market daily to make life easier and more convenient. Professor K sometimes thought about inventing good, useful products. He pursued some of his ideas to make them reality. That is what inventors do; they make their dreams reality. Even if you do not become an inventor, the dreams you have for life can become reality if you work hard and do not lose your vision. Inventions are always the result of dreams.

This lesson is a continuation of Lesson 8, with the addition of the numbers 7-12 included in the multiplication table. The following multiplication facts, as well as those in Lesson 8, should be memorized. This will make multiplication, and math in general, much easier. The numbers to the left of an equal sign are factors of the product to the right of the equal sign. The same rules and principles apply for these multiplications as in Lesson 8. Professor K reminds you that you will be required to memorize and quote the multiplication facts from Lessons 8 and 9 before you proceed to Lesson 10.

## VOCABULARY

***Inventions:*** things created that had not previously been made
***U.S. Patent Office:*** place where descriptions of inventions are recorded by inventors and kept in government files
***Red Cross:*** an international organization that helps people who suffer from disasters such as wars, earthquakes or severe weather

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

**Table 9.1**

| | | |
|---|---|---|
| 7 × 1 = 7 | 8 × 1 = 8 | 9 × 1 = 9 |
| 7 × 2 = 14 | 8 × 2 = 16 | 9 × 2 = 18 |
| 7 × 3 = 21 | 8 × 3 = 24 | 9 × 3 = 27 |
| 7 × 4 = 28 | 8 × 4 = 32 | 9 × 4 = 36 |
| 7 × 5 = 35 | 8 × 5 = 40 | 9 × 5 = 45 |
| 7 × 6 = 42 | 8 × 6 = 48 | 9 × 6 = 54 |
| 7 × 7 = 49 | 8 × 7 = 56 | 9 × 7 = 63 |
| 7 × 8 = 56 | 8 × 8 = 64 | 9 × 8 = 72 |
| 7 × 9 = 63 | 8 × 9 = 72 | 9 × 9 = 81 |
| 7 × 10 = 70 | 8 × 10 = 80 | 9 × 10 = 90 |
| 7 × 11 = 77 | 8 × 11 = 88 | 9 × 11 = 99 |
| 7 × 12 = 84 | 8 × 12 = 96 | 9 × 12 = 108 |
| 10 × 1 = 10 | 11 × 1 = 11 | 12 × 1 = 12 |
| 10 × 2 = 20 | 11 × 2 = 22 | 12 × 2 = 24 |
| 10 × 3 = 30 | 11 × 3 = 33 | 12 × 3 = 36 |
| 10 × 4 = 40 | 11 × 4 = 44 | 12 × 4 = 48 |
| 10 × 5 = 50 | 11 × 5 = 55 | 12 × 5 = 60 |
| 10 × 6 = 60 | 11 × 6 = 66 | 12 × 6 = 72 |
| 10 × 7 = 70 | 11 × 7 = 77 | 12 × 7 = 84 |
| 10 × 8 = 80 | 11 × 8 = 88 | 12 × 8 = 96 |
| 10 × 9 = 90 | 11 × 9 = 99 | 12 × 9 = 108 |
| 10 × 10 = 100 | 11 × 10 = 110 | 12 × 10 = 120 |
| 10 × 11 = 110 | 11 × 11 = 121 | 12 × 11 = 132 |
| 10 × 12 = 120 | 11 × 12 = 132 | 12 × 12 = 144 |



Study the following example using the multiplication facts for the numbers *7-12*. In this example, Professor K shows you how to find the answer to the following problem.

**Example Set 1**

Seven ***Red Cross*** volunteers worked *41* hours each to help victims rescued from the rubble of the Twin Towers. How many hours did the volunteers work altogether?

***Step 1***

41.
× 7.

In this problem you are asked to multiply *41* by seven. Adding *41* to itself *7* times would produce the answer; however, multiplication would produce the answer faster. (See Step 1.)

***Step 2***

41.
× 7.
7.

To begin to work a multiplication problem such as this, the numbers in the *ones* column are multiplied together first. In this example *7 × 1 = 7*. The *7* is placed in the *ones* position of the answer. (See Step 2.)

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

**Step 3**

$$\begin{array}{r} 41. \\ \times\ 7. \\ \hline 287. \end{array}$$

The next step to solve this multiplication problem is to multiply *7* times *4* (*7 × 4 = 28*). The answer to this smaller multiplication problem is *28*. The *8* is placed in the product (answer) in the *tens* column and the *2* is now carried to the *hundreds* column. Since there is no other number in the *hundreds* place, the *2* is placed in the answer as shown. This makes the product of *41 × 7 = 287*. (See Step 3.)

---

Practice your multiplication skills by working the activities for this lesson.

# LIFE PRINCIPLE



"TELL ME WITH WHOM YOU GO, AND I'LL TELL YOU WHAT YOU ARE."

—Irish Proverb

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.




©Copyright 2005, Paradigm Accelerated Curriculum

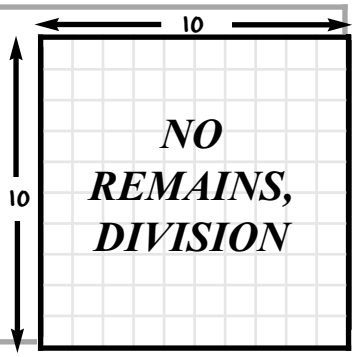
DUPLICATION BY WRITTEN PERMISSION ONLY.




Lesson 10

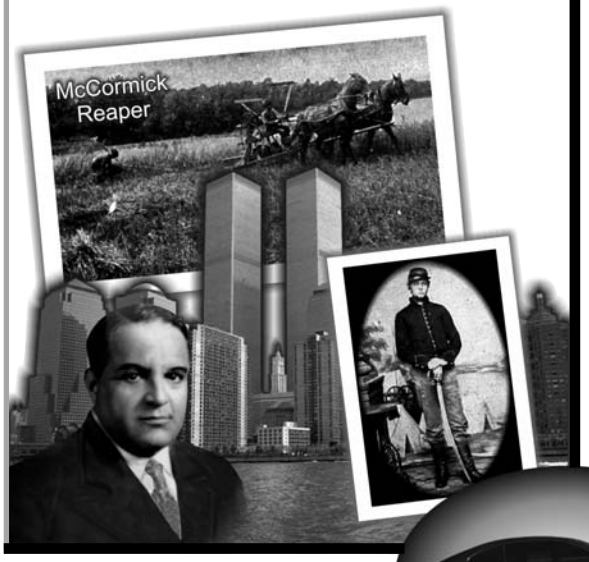
*Fundamentals*

# Chapter 1




## Section 2

***Lessons 6-10***






As Professor K gained experience driving his car, he was able to go faster and feel more confident behind the wheel. You, too, can gain confidence and ability when driving your mathematical vehicle. You put the mathematical "car" in drive (forward) when you add, and in reverse when you subtract. In Lessons 8 and 9 you put that mathematical "car" into high gear with multiplication. In this lesson you will go in reverse again, but much more rapidly.

***Division*** is a shortcut to subtraction and is the opposite or reverse operation from multiplication. In order to do division, the thought must be, "Into how many equal parts can I divide the number?"

Study the principle of mathematics below to gain an understanding of the meaning of division.



**DEFINITION: DIVISION**

*Division* is the process or operation by which a number is split up or divided into as many equal parts as required.

## VOCABULARY

***Division:*** a shortcut to subtraction, the opposite operation to multiplication
***Quotient:*** the answer to a division problem
***Dividend:*** the number *inside the tabletop* ($\overline{)6}$) or *first in a sentence style* division problem (6 ÷ 2)
***Divisor:*** the number *outside the tabletop* or *last in a sentence style* division problem

**Example Set 1**

- 8 ÷ 4 = 2 *or* 8 = 2 + 2 + 2 + 2
  8 is divided into 4 equal parts of size 2. $4\overline{)8}$ $\;2$
- 20 ÷ 5 = 4 *or* 20 = 4 + 4 + 4 + 4 + 4
  20 is divided into 5 equal parts of size 4. $5\overline{)20}$ $\;4$
- 6 ÷ 3 = 2. *2* is the ***quotient***. $3\overline{)6}$ $\;2$
- 25 ÷ 5 = 5. *25* is the ***dividend***. $5\overline{)25}$ $\;5$
- 12 ÷ 4 = 3. *4* is the ***divisor***. $4\overline{)12}$ $\;3$

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

Read the following examples of *short division* carefully to gain a better understanding of the process of division. Short division answers can usually be arrived at mentally if you know your multiplication tables, and can usually be completed in 3 easy operations: divide, multiply, subtract.



**Example Set 2**

***Step 1*** $2\overline{)6}$

The problem is asking you to find the answer to *6* divided by *2*. To find the answer, or quotient, you must ask yourself, "How many times can I *divide* six into equal parts by *2*s"? (See Step 1.)

***Step 2*** $\begin{array}{r} 3 \\ 2\overline{)6} \end{array}$

Looking at *6*, you can see that it is *3 + 3*. Therefore, *6* can be divided equally, *2* times, by three. In other words *6 ÷ 2 = 3*, therefore the quotient, 3, is placed on top of the division symbol directly above the 6, which is called the ***dividend***. (See Step 2.)

***Step 3*** $\begin{array}{r} 3 \\ 2\overline{)6} \\ -6 \\ \hline 0 \end{array}$

Next, we must *multiply* the quotient (3) times the ***divisor*** (2). The product (answer) is 6 and is written under the *6* in the problem. Next, we *subtract* and the result (*difference*) is zero, *0*, written as shown. (See Step 3.)

**Example Set 3**

***Step 1*** $4\overline{)32}$

Look at the example *32 ÷ 4*. The question must be asked, "How big are the parts if I *divide 32* into *4* equal parts?"

***Step 2*** $\begin{array}{r} 8 \\ 4\overline{)32} \end{array}$

Four will not go into *3*, but *4* will go into *32* evenly. If you divide *32* into *4* equal parts, the parts are of a size *8*, or *8 + 8 + 8 + 8 = 32*. (See Step 2.) Since *4* will divide into *32* eight times, *8* is the quotient of the division problem and you write it on top of the division symbol as shown (See Step 2.) and above the last digit of the dividend (the number being divided). We are not dividing *4* into *3*, so the quotient does not go above the 3. We are dividing *4* into *32*, so the answer is placed above the last digit of the *32*.

***Step 3*** $\begin{array}{r} 8 \\ 4\overline{)32} \\ -32 \\ \hline 0 \end{array}$

As in **Example Set 2**, we *multiply* the quotient (*8*) times the divisor (*4*). The product (answer) of *32* is written under the *32* in the problem. Next, we subtract. The result (*difference*) is zero, *0*. (See Step 3.)

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

Professor K continues the study of division by conveying to you the accomplishments of Dr. Ivan Panin. Dr. Panin discovered a mathematical design of the Greek text of the New Testament and the Hebrew text of the Old Testament. Hebrew and Greek cultures use letters of the alphabet for numerical values. This means that each word has a numeric (number) value equal to the sum of the values of each letter. Dr. Panin's discoveries are exemplified in the Bible's Greek text of the first chapter of Matthew, verses 1-17. In these verses Dr. Panin found that when the words are compared to each other, the number seven or its multiples occur frequently.

The word "the" occurs 56 times and the passage contains 56 nouns ($56 = 7 \times 8$). This portion of Matthew can be divided into two main sections, 1-11 and 12-17. The first section contains 49 ($7 \times 7$) different Greek words. Dr. Panin looked at these 49 words and noticed that the number of words beginning with vowels were 28 and with consonants, 21. The total number of letters in the Greek words of this passage and the number of total vowels and consonants were also multiples of the number seven. The number of common and proper nouns was also a multiple of seven.

The number seven or its multiples are interlaced perfectly throughout the Greek words. Professor K will show you more division of numbers and more of the discoveries of Dr. Panin from Matthew in **Example Sets 4 and 5**.

Read the following *long division* examples carefully. Long division problems are a little more involved; but, as in larger multiplication problems, are nothing more than several smaller division problems in one larger problem. However, two additional operations are needed: *compare* and *bring down*.

**Example Set 4**

***Step 1***

$7\overline{)266}$

The number of letters in the *49* words of the first section of Matthew 1:1-17 is *266*. Show that *266* is a multiple of *7*. (Seven must divide *266* evenly without a remainder.)

***Step 2***

$$\begin{array}{r} 038 \\ 7\overline{)266} \\ \underline{-21}\downarrow \\ 56 \\ \underline{-56} \\ 0 \end{array}$$

In this problem the first number to consider is *2*. You cannot divide *7* into *2*, so a zero, *0*, can be placed above the 2 to mark off that place in the quotient row. The next number, *6*, is combined with the *2* to make *26: 7* will *divide* into *26* because *26* contains *3* sevens, therefore a *3* is placed in the quotient, directly over the first *6* in the dividend. Then the quotient (*3*) is *multiplied* times the divisor (*7*). The product (answer) is written under the *26* in the problem and subtracted. (See Step 2.) This difference is *5*. *Compare* the difference (*5*) with the divisor (*7*). The difference must always be smaller. If it is the same size or larger the quotient is too small and your final answer will be wrong. Now you need to *bring down* the next number (*6*) in the dividend (*266*) beside the *5* to make *56*. Now we go back through the five steps of division until the problem is finished. So, the next division is $56 \div 7$. The number *7* divides *56* into *8* equal parts ($56 \div 7 = 8$ or $7 \times 8 = 56$). Place the *8* in the quotient row to the right of the *3*. Multiply the quotient (*8*) times the divisor (*7*). The product, *56*, is subtracted from the *56* in the problem and the difference is zero. (See Step 2.) The quotient of *266* divided by *7* is *38*. *266* is a multiple of *7*, because *7* divides *266* evenly.

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

**Example Set 5**

**Step 1**

$$28\overline{)42364}$$

The numerical value of the *72* words is *42,364*. Show that *42,364* is a multiple of *7*. (Seven must divide *42,364* evenly, without a remainder.) Professor K will show division using a two-digit number and therefore uses *28* as the divisor. The number *28* is a multiple of *7 (7 × 4 = 28)*.

**Step 2**

$$\begin{array}{r} 015__ \\ 28\overline{)42364} \\ -28\downarrow \\ \hline 143 \\ -140 \\ \hline 3 \end{array}$$

In this problem the first number to consider is *4*. You cannot *divide 28* into *4*, so a zero, *0*, can be placed above the *4* to mark off that place. (See Step 2.) The next number, *2*, is combined with the *4* to make *42*. *42* does contain one *28*, therefore a quotient of one is placed over the *2* in the dividend. Then we *multiply 1 × 28*. We *subtract* the product (*28*) from *42*. The difference is *14*. Now, *compare* this with the divisor (*28*). *14* is smaller, so we can proceed. Next, you need to *bring down* the next number (*3*) in the dividend (*423*) beside *14* to make *143.* (See Step 2.) Now, the process begins again with the *division* of *143 ÷ 28*. The number *28* divides *143* into *5* equal parts, or *28 × 5 = 140*. Therefore you place the *5* in the quotient row to the right of the *1*. The quotient (*5*) is *multiplied* times the divisor (*28*). The product, *140*, is *subtracted* from *143* and the difference is *3.* (See Step 2.) Now *compare* this difference (*3*) with the divisor (*28*). *3* is smaller so we can proceed.

**Step 3**

$$\begin{array}{r} 01513 \\ 28\overline{)42364} \\ -28\downarrow\ \downarrow\ \downarrow \\ \hline 143 \\ -140\downarrow\ \downarrow \\ \hline 36 \\ -28\ \downarrow \\ \hline 84 \\ -84 \\ \hline 0 \end{array}$$

The next number in the dividend (*6*) is *brought down* beside the *3* to make *36*. In repeating the process, *28 divides* into *36* one time; so a *1* is placed above the next number in the dividend (*6*) in the quotient row, as shown in Step 3. The quotient (*1*) is multiplied times the divisor (*28*). The product, *28*, is *subtracted* from *36*. The difference is *8*. *Compare 8* to *28*. The *8* is smaller so we can proceed. The next number in the dividend (*4*) is *brought down* and combined with the *8* to make *84*. The process repeats as the number *84* is *divided* by *28* to obtain *3* as the quotient. The *3* is placed above the *4* in the dividend as shown in Step 3. The quotient (*3*) is *multiplied* times *28*. The product (*84*) is *subtracted* from the *84* in the problem to obtain a difference of zero. (See Step 3.) So we see that *28*, a multiple of *7*, divides *42,364* evenly.

Professor K points out that in this section, the divisions are even, therefore they have nothing left over (no remainder). The next lesson teaches you about answers with remainders but don't forget to do the activities for this lesson.



©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

# LIFE PRINCIPLE



**"A man without life purpose divides his energy and multiplies his problems."**

—Ronald Johnson

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.




©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

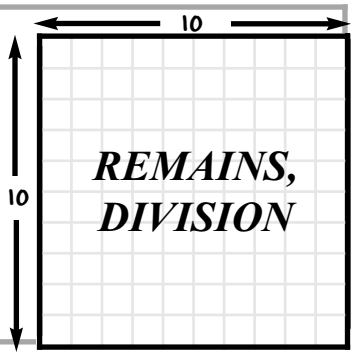
# Basic Math Skills




Lesson 11

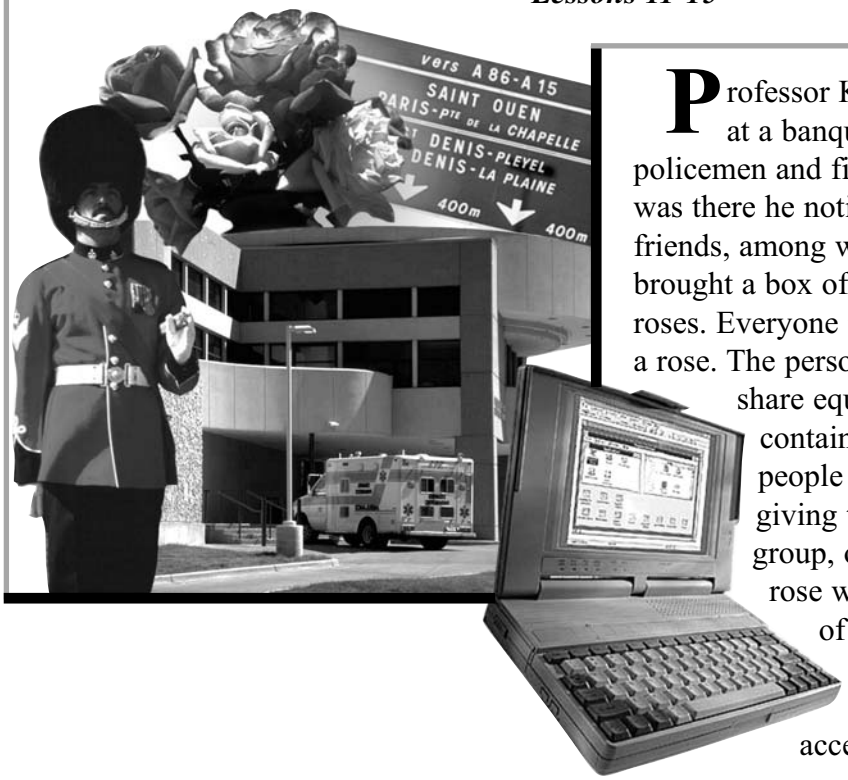
*Fundamentals*

# Chapter 1




## Section 3

***Lessons 11-15***




Professor K was asked to speak at a banquet to honor policemen and firefighters. While he was there he noticed a group of friends, among whom someone had brought a box of long-stemmed roses. Everyone in the group wanted a rose. The person with the box was willing to share equally with her friends. The box contained fifteen roses, but seven people were in the group. After giving two roses to everyone in the group, one rose remained. The one rose was the ***remainder.*** The owner of the box of roses offered Professor K the remaining rose, which he graciously accepted.



**Professor K**

### Definition: Remainder

A remainder is what is left over (remains) after a number has been divided into as many equal parts as required.

## VOCABULARY

***Remainder:*** what is left over (remains) after a number has been divided into as many equal parts as required.
***Quotient:*** the answer to a division problem

- 5 divided by 2 = 2 with a remainder of 1. Since 2 × 2 = 4, then 1 is left over from 5 - 4.
- 17 divided by 5 = 3 with a remainder of 2, since 3 × 5 = 15, and 17 - 15 = 2.

Many division problems have remainders. This lesson will discuss how to work division problems involving remainders.

Professor K will show you in the following examples how to work such problems and how to write the ***quotients***.

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

**Example Set 1**

**Step 1** $3\overline{)49}$

In this example, *49* is divided by *3*. (See Step 1.) To begin the process, look at the first number (*4*) in the dividend (*49*).

**Step 2**

$$\begin{array}{r} 16 \\ 3\overline{)49} \\ -\,3\downarrow \\ \hline 19 \\ -\,18 \\ \hline 1 \end{array}$$

Three will *divide* into *4* one time, so a *1* is placed in the quotient row above the *4* in the dividend. (See Step 2.) The quotient (*1*) is *multiplied* times the divisor (*3*). The product (*3*) is *subtracted* from the *4*, leaving a difference of *1*. *Compare* the difference (*1*) with the divisor (*3*). The *1* is smaller so we can proceed. The *9* of *49* is *brought down* beside the *1* to make *19*. Your objective is to get a product that is as close to *19* as possible without having a product greater than the dividend. $3 \times 6 = 18$ and $3 \times 7 = 21$. *18* is closer to *19* than *21* is, so *6* is written in the quotient row. The product, *18*, is written under *19* and subtracted. In this case the difference is *1*. The remainder is *1* because no numbers are left to bring down. (See Step 2.)

The quotient may be written one of two ways: 16 r1 or $16\frac{1}{3}$

**Example Set 2**

**Step 1** $12\overline{)149}$

Here, *149* is divided by *12*.

**Step 2**

$$\begin{array}{r} 12 \\ 12\overline{)149} \\ -\,12\downarrow \\ \hline 29 \\ -\,24 \\ \hline 5 \end{array}$$

Upon a closer look, you can see that *12* will *divide* into *14* one time, or we can say there is one *12* in *14*. Therefore, *1* is placed in the quotient row above the *4* of *149*. The *multiplication* of $1 \times 12 = 12$. The product (*12*) is placed under the 14 and *subtracted*, leaving a difference of *2*. *Compare*. Two is less than *12*, so we can proceed. Now *bring down* the *9* beside the *2*, to make *29*. *29 divided* by *12* shows that two *12s* are in *29* ($2 \times 12 = 24$). The *2* is placed beside the *1* in the quotient row. The *multiplication*, $12 \times 2 = 24$, is performed and *24* is placed under the *29* and *subtracted*. The difference is *5*. Because no numbers are left to bring down, the remainder is *5*. (See Step 2.)

The answer is written one of two ways: 12 r5 or $12\frac{5}{12}$.

Professor K encourages you to complete the activity questions to demonstrate your knowledge of division.

## LIFE PRINCIPLE



**"Ability is what you're capable of doing . . . . Motivation determines what you do . . . . Attitude determines how well you do it."**

—Anonymous

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

# Basic Math Skills

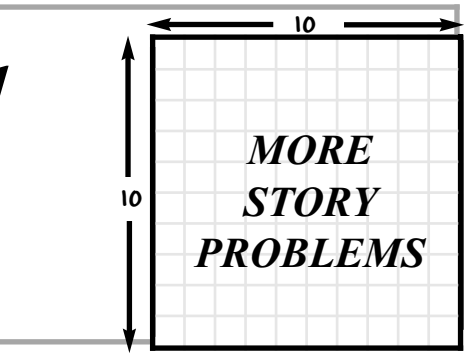
# Chapter 1

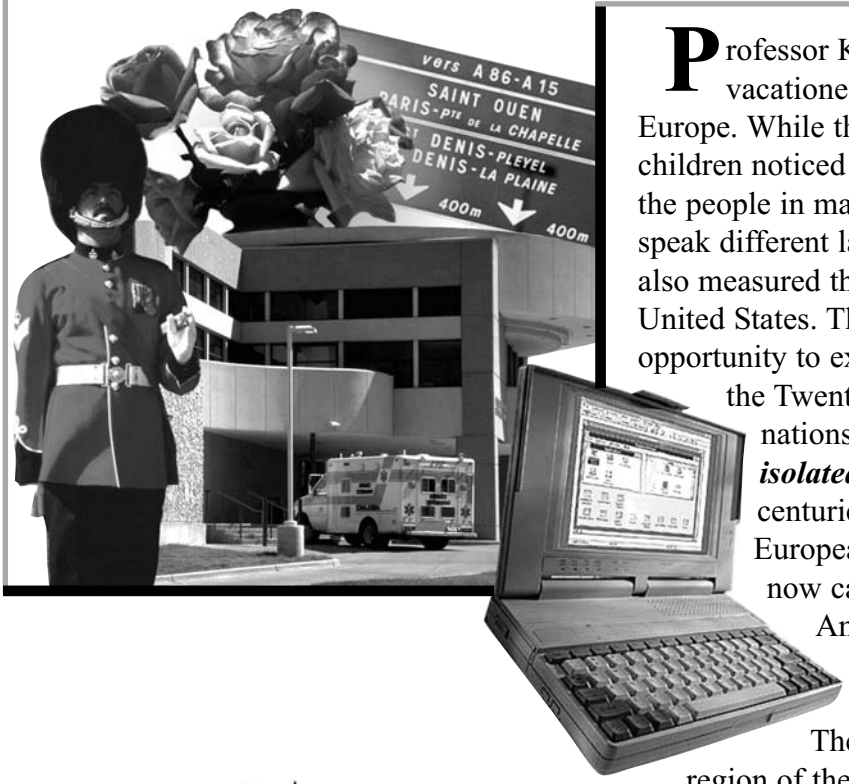
Lesson 12

*Word Problems*

## Section 3

***Lessons 11-15***






**Professor K**

Professor K and his family vacationed a few years ago in Europe. While there, Professor K's children noticed that not only did the people in many parts of Europe speak different languages, but they also measured things differently than in the United States. This gave Professor K the opportunity to explain to his children that until the Twentieth Century, the peoples and nations of the world lived somewhat ***isolated*** from each other. Several centuries opened and closed before Europeans learned of the continents now called North and South America. Time ***elapsed***, but travel among various areas of the world was still difficult. Therefore, what happened in one region of the world had little effect on other parts of the world.

In the 1900s, however, life changed around the globe. Wars in the past had been confined to certain areas or regions, as had ***economic depressions***. In the 1900s, depressions and wars became worldwide problems. The world was affected dramatically by World War I (WWI), World War II (WWII), and the terrorists' attacks on the United States in 2001. The invention of computers and faster, easier, cheaper means of transportation in a sense made the world "smaller." What happens in Japan's stock market immediately affects other markets around the world and ***vice-versa***. Nations depend on trade with other nations of the world in order to balance their budgets.

***Contemporary*** people in international business know and understand various methods used by the countries of the world to measure things, communicate and respond to emergencies.

## VOCABULARY

***Isolated:*** separated
***Elapsed:*** passed, went by, expired
***Economic depressions:*** long periods of inactivity in business and trade with widespread unemployment.
***Vice-versa***: switched about, change of order
***Contemporary:*** modern, current, present time

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

In Section 3, Professor K will show you how to solve multiplication and division word problems. Also, you will learn the basics of the two major systems used in the world for measurement: the English System (sometimes referred to as the U. S. System) and the Metric System.

## Multiplication and Division Word Problems

You will need to recall or refer to the **Steps to solving a word problem** from Lesson 7. These steps should be followed to solve any word problem. In this lesson the story problems will involve ideas which will require multiplication to solve each problem.



### Definition: Average

average = total divided by how many (quantity)



Calculating an average is a two-step process:

1. Add all the numbers to get a total;
2. Divide the total by the quantity of numbers added together.

**Example Set 1**

Calculate the average of the following numbers:

a) 50, 30, 10. Add: 50 + 30 + 10 = 90. How many numbers?: 3 Divide: 90 ÷ 3 = 30

b) 32, 64. Add: 32 + 64 = 96. How many numbers?: 2 Divide: 96 ÷ 2 = 48

**"Key" words used and what they mean in word problems**

| | |
|---|---|
| product | multiplication problem |
| multiple, or multiplied by | multiplication problem |
| times | multiplication problem |
| quotient | division problem |
| divided by | division problem |
| average | addition and division problem |

**Example Set 2**

Mary worked for a hospital for 4 weeks (5 days per week) and received $60 per day. How much money did Mary earn at the end of the job?

Following the steps in **Steps to solving a word problem** (Chapter 1, Lesson 12), the problem is worked by the following procedure:

a) Read the problem to see that Mary had a job and worked a certain period of time for a certain amount of money. The problem is to calculate the amount of money Mary earned.

b) Rereading the problem, you see that Mary worked 4 weeks (5 days per week) and was paid $60 per day.

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

c) The wording of the problem shows that the operation needed to solve the problem is multiplication (see **words used** section on previous page) because of multiple days of work for which Mary was paid multiples of dollars.

d) Multiply:

| | | | |
|---|---|---|---|
| 4 | weeks | 60 | dollars per day |
| × 5 | days/week | × 20 | total days Mary worked |
| 20 | total days worked | $1,200 | total amount of money Mary earned |

e) Check to see if the answer looks reasonable.

---

**Example Set 3**

Four times eleven less six is what number?
Following the steps in **Steps to solving a word problem**, you work the problem by the following procedure:

a) Read the problem to see that two numbers are multiplied together ("times"), and another number is subtracted ("less") from the product in order to answer the question.

b) Rereading the problem, you can see that the two numbers multiplied together are 4 and 11; 6 is subtracted from their product.

c) The wording of the problem tells you that the operations needed to solve the problem are multiplication and subtraction, since the words "times" and "less" appear in the wording.

d) Multiply: Subtract:

| | |
|---|---|
| 4 | 44 |
| × 11 | - 6 |
| 44 | 38 |

e) Check to see if the answer looks reasonable.

---

**Example Set 4**

Twenty-four divided by eight less one produces what number?
Following the steps in **Steps to solving a word problem**, the problem is worked by the following procedure:

a) Read the problem to see that one number is "divided by" another number (a division problem), and another number is subtracted ("less") from the quotient in order to answer the question.

b) Rereading the problem, you can see that the two numbers involved in the division are 24 and 8, and you will need to subtract 1 from the quotient.

c) Wording of the problem informs you that the operations needed to solve the problem are division and subtraction since the words "divided by" and "less" appear in the wording.

d) Divide: $24 \div 8 = 3$ Subtract: 3 - 1 to get 2 as the final answer to the problem

e) Check to see if the answer looks reasonable.

---

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

**Example Set 5**

Sixty times three increased by four is what number?
Following the steps in **Steps to solving a word problem**, the problem is worked by the following procedure:

a) Read the problem to see that one number is "times" another number (a multiplication problem), and is "increased by" (added to) other numbers.
b) Reread to see that the two numbers involved in the multiplication process are 60 and 3, and that 4 more are added.
c) The wording of the problem informs you that the operations needed to solve the problem are multiplication ("times") and addition ("increased by").
d) Multiply: $60 \times 3 = 180$ Add: $180 + 4 = 184$
e) Check to see if the answer looks reasonable

---

**Example Set 6**

Robert attended firefighter's school where he took four math tests. His scores were 80%, 94%, 72% and 62%. What was Robert's average for the class?
Following the steps in **Steps to solving a word problem**, the problem is worked by the following procedure:

a) Read the problem to see that Robert has taken 4 tests (scores given), and you are to find the average. This means you will have to add the 4 test scores and then divide by 4 (the number of tests).
b) Rereading the problem, you can see that the tests scores are 80, 94, 72 and 64. The total number of tests is 4.
c) The wording of the problem tells you that the operations needed to solve the problem are addition and division because of the word, "average".
d) Add

$$\begin{array}{r} 80 \\ 94 \\ 72 \\ +\ 62 \\ \hline 308 \end{array}$$

Now divide 308 by 4 (the number of tests). Notice the equation, $308 \div 4 = 77$. So Robert's test average is 77%.

e) Check to see if the answer looks reasonable.

---

Practice your problem solving skills by working the activities for this lesson.

# LIFE PRINCIPLE



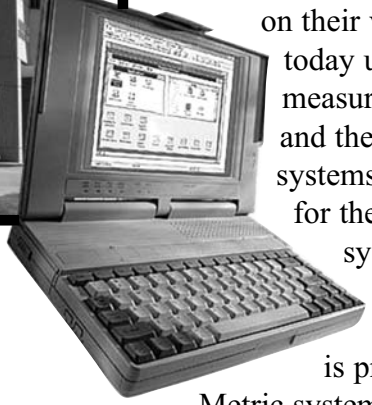
"Glass and reputation are easily cracked, and never well mended."
—Benjamin Franklin

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

# Basic Math Skills

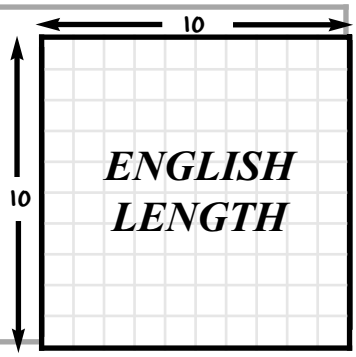


Lesson 13

*Converting*

# Chapter 1




## Section 3

***Lessons 11-15***



**Professor K**

You use mathematics for more than just adding, subtracting, multiplying and dividing. Mathematics is useful for measuring ***length***, volume and weight, which will be discussed in Lessons 13 through 15.

As Professor K's family discovered on their vacation to Europe, the world today uses two different forms for measurement, the ***English System*** and the ***Metric System***. Both of these systems are presented in this section for the measuring of length in both systems and volume in the English system. The English system for measuring length is presented in this lesson. The Metric system for length is discussed in Lesson 14. The English system for volume is given in Lesson 15.

Professor K's children asked how to determine different units when measuring lengths. Professor K explained that before we can measure something, we must decide what unit of measurement is needed or desired. In ancient times, wise men developed an advanced standard unit to measure the length of objects, such as the pyramids of Egypt and Mexico. Men used their hands, feet, and arms for some semblance of a standard. However, not all people had the same sized hands, feet, or arms so the crude system was lacking a necessary standard or accuracy.

Professor K asked his children if they remembered their mother measuring a *yard* of material by holding one end in her fingers and touching the other end to her nose. The length of a *yard* was once defined as the length of the outstretched arm of an adult. The *foot* was defined as the length of an adult human foot (thereby the word, "foot," for 12 inches).

## VOCABULARY

***Length:*** identifies how long something is
***English System:*** system of measurement used primarily in the United States
***Metric System:*** system of measurement used by nearly every country of the world
***Convert:*** a method of changing one unit of measurement into another unit of measurement

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

The English developed a standard measurement system which was called the English System of Measurement. The American Colonies adopted the English system. Today, the United States is primarily the only country in the world using the English System of measurement; therefore this system is also referred to as the *U.S. System*.

Professor K's family was still confused about the different units and how to change or ***convert*** them into other units. Professor K wrote **Table 13.1** on a piece of paper. **Table 13.1** shows the terminology and conversions used in the English System for measuring length.

**Table 13.1**

**English System**
**Length Equivalents**

| | | |
|---|---|---|
| 12 inches | = | 1 foot |
| 3 feet | = | 1 yard |
| 5.5 yards | = | 1 rod |
| 320 rods | = | 1 mile |
| 1,760 yards | = | 1 mile |
| 5,280 feet | = | 1 mile |

Professor K then explained that to change or convert one unit to another unit, the information in **Table 13.1** is very useful. Study the following examples to see how Professor K explained to his family how to do unit conversions in the English System. You need to memorize the facts in **Table 13.1** so solving problems involving the English system of measuring length will be quicker and easier.

**Example Set 1**

| | | |
|---|---|---|
| ***Step 1*** | 36 inches = ?? feet | To convert or change *36* inches into feet, remember the conversion from **Table 13.1** that shows 12 inches = 1 foot. With this fact in mind, the problem of "How many feet are in *36* inches?" is one of division. |
| ***Step 2*** | 36 ÷ 12 = 3 | To change *36* inches into feet, divide *36* inches by *12* inches, because there are 12 inches in one foot. (See Step 2.) |
| ***Step 3*** | 36 inches = 3 feet | Therefore, *36* inches = *3* feet. |



©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

As you look again at **Table 13.1**, notice that when changing the units in the left column to units in the right, multiplication solves the problem. When converting units from the right column to the left column, use division to solve the problem. **Table 13.2** shows this procedure more clearly.

**Table 13.2**

| **Multiply** (from larger to smaller units) | | **Divide** (from smaller to larger units) | |
|---|---|---|---|
| feet, ft | → inches, in | feet, ft | ← inches, in |
| yards, yds | → feet, ft | yards, yds | ← feet, ft |
| rods | → yards, yds | rods | ← yards, yds |
| mile, mi | → rods | mile, mi | ← rods |
| mile, mi | → yards, yds | mile, mi | ← yards, yds |
| mile, mi | → feet, ft | mile, mi | ← feet, ft |

**Example Set 2**



While in the police training academy, James hiked 6 miles on a 3-day training exercise. He wanted to know how many yards he had walked.

To find the answer, James will have to recall the measurements from **Table 13.2** and remember that to change miles (the larger unit) to yards (the smaller unit), he will have to multiply. Therefore, the problem will be:

1760 yards
× 6 miles
10,560 yards walked

So James has calculated that he hiked 10,560 yards on his 3-day training exercise.

Sometimes you must make more than one conversion in order to get the desired answer, as in **Example Sets 3 and 4**.

**Example Set 3**

***Step 1*** 2 miles = ??? inches

This problem is asking, "How many inches are in *2* miles?" Since **Table 13.1** does not provide a straightforward conversion, other calculations must be done in order to make the change from miles to inches.

***Step 2*** 1 mile = 5,280 feet

5280
× 2
10560

From **Table 13.1**, notice that 1 mile contains 5,280 feet; therefore, when multiplied by *2* miles, the total number of feet is *10,560* feet. (See Step 2.)

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

**Step 3**

1 foot = 12 inches

$$\begin{array}{r} 10560 \\ \times\ 12 \\ \hline 21120 \\ +\ 105600 \\ \hline 126720 \end{array}$$

Refer once again to **Table 13.1**. Notice that a foot contains 12 inches. Now you multiply the answer from the first conversion, *10,560*, and by *12*. (See Step 3.)

The conversion then for 2 miles to inches is: 2 miles = 126,720 inches

---

**Example Set 4**

**Step 1** 5 miles = ?? rods

This problem is asking, "How many rods are in 5 miles?" **Table 13.1** gives the number of *yards* in a rod. Since 320 rods = 1 mile, 5 miles will be multiplied by *320*.

**Step 2**

320 × 5 =
320 × 5 = 1600

Therefore, *5* miles = *1600* rods.

---

Apply what you have learned about English measurements by working the activities for this lesson.

# LIFE PRINCIPLE



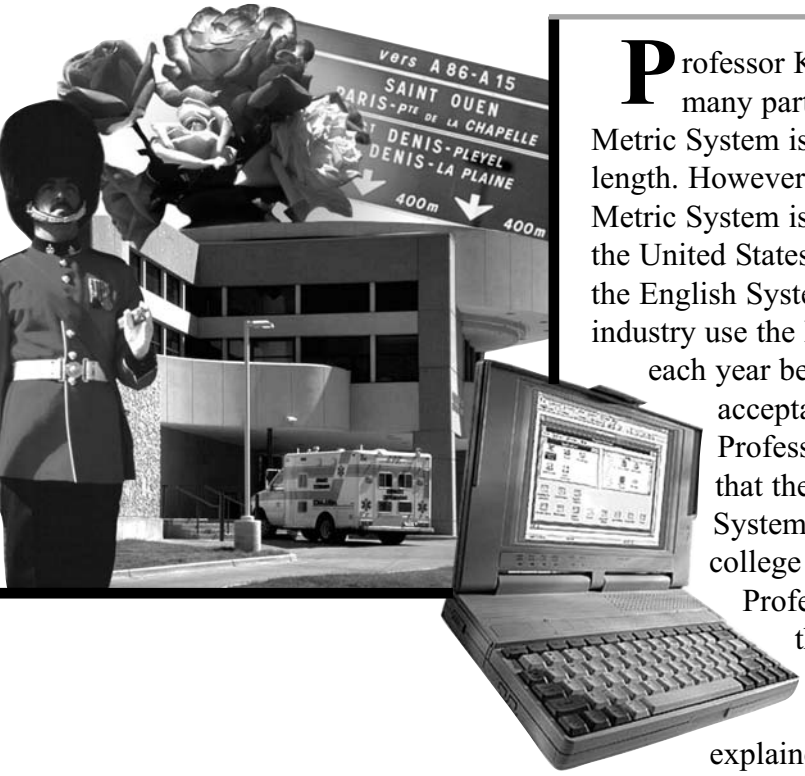
"Our deeds determine us, as much as we determine our deeds."

—George Eliot

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

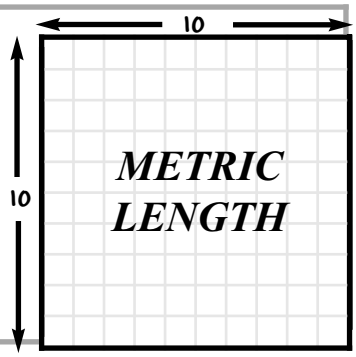
# Basic Math Skills




Lesson 14

*Converting*

# Chapter 1




## Section 3

***Lessons 11-15***



**Professor K**

Professor K told his family that in many parts of the world the Metric System is used to measure length. However, the use of the Metric System is increasing even in the United States, which depends on the English System. Science and industry use the Metric System more and more each year because of the worldwide acceptance of the Metric System. Professor K explained to his children that they needed to learn the Metric System in order to prepare for college and a career.

Professor K's children remarked that the Metric system had "funny" words in its ***terminology***. Professor K explained that ***prefixes*** are used to describe different lengths in the metric system. Different prefixes, or "word beginnings," are added to the word, ***meter***. **Table 14.1** shows these prefixes and their meanings.

## VOCABULARY

***Terminology:*** proper use of words as names or symbols
***Prefixes:*** letters placed in front of words to add meaning
***Meter:*** the basic unit of measurement for length in the metric system
***Symbol:*** letters or numbers used to represent something (an abbreviation)

**Table 14.1**

| Prefix | Abbreviation | Meaning |
|---|---|---|
| kilo- | k | × 1000 |
| hecto- | h | × 100 |
| deka- | dk | × 10 |
| unit | | × 1 |
| deci- | d | × $^{1}/_{10}$ |
| centi- | c | × $^{1}/_{100}$ |
| milli- | m | × $^{1}/_{1000}$ |

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

The first and second columns of **Table 14.1** show prefixes and the abbreviations in decreasing size, beginning with the largest and going to the smallest numbers. The third column shows the meaning of each ***symbol***. Memorizing the prefixes, their abbreviations and meanings, will be of extreme value to you when working with the *Metric System of measurement.*

**Table 14.2**

| **abbreviation:** | **km** | **hm** | **dkm** | **m** | **dm** | **cm** | **mm** |
|---|---|---|---|---|---|---|---|
| **meaning:** | × 1000 | × 100 | × 10 | × 1 | × $^1/_{10}$ | × $^1/_{100}$ | × $^1/_{1000}$ |

Multiply (from larger to smaller units) ← → Divide (from smaller to larger units)

Under the abbreviations and meanings in **Table 14.2,** arrows show that when changing prefixes from left to right, each step requires you to multiply by multiples of 10. **Table 14.2** also shows that when changing prefixes right to left, each step requires you to divide by multiples of 10. This procedure is called the *Directional Method of Converting.* The following examples help explain this procedure.

**Example Set 1**

a) 1 kilometer = ____ meters
1 × 1000 = 1000 meters
1 km × 10 = 10 hm
10 hm × 10 = 100 dkm
100 dkm × 10 = 1000 m

Here the change from kilometers to meters is simple since kilo- means *1000* times; the *1* is multiplied by *1000*, to get *1000* meters as the answer. To change kilometers to meters using the *Directional Method of Converting*, the change requires moving from **km** to **m**, or left to right, therefore, you must multiply by ten *3* times, because you must make three prefix moves from **km** to **m**. (**km** $\xrightarrow{1}$ **hm** $\xrightarrow{2}$ **dkm** $\xrightarrow{3}$ **m**) Refer to **Table 14.2**. Notice you are moving left to right from **km** to **m**, therefore you multiply by 10, three times (10 × 10 × 10 = 1000).

b) 2500 mm = _____ dm
2500 mm ÷ 10 = 250 cm
250 cm ÷ 10 = 25 dm

In this example the move you will make is right to left from **mm** to **dm**, which means division. Using the *Directional Method of Converting* you can see by looking at Table 14.2 that 2 prefix moves are necessary to go from **mm** to **dm**. This gives an answer of 25 dm. (**dm** $\xleftarrow{2}$ **cm** $\xleftarrow{1}$ **mm**) Refer to **Table 14.2**. Notice you are moving right to left from **mm** to **dm**, therefore you divide by 10, two times.

Demonstrate your understanding of the Metric System by working the activities for this lesson.

## LIFE PRINCIPLE



"Nothing worthwhile is ever easy."
—Anonymous

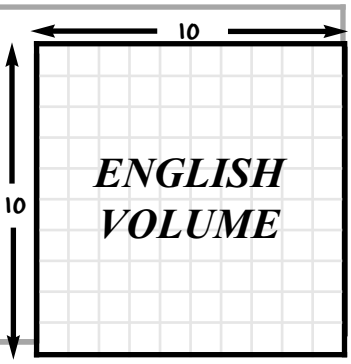
©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

# Chapter 1

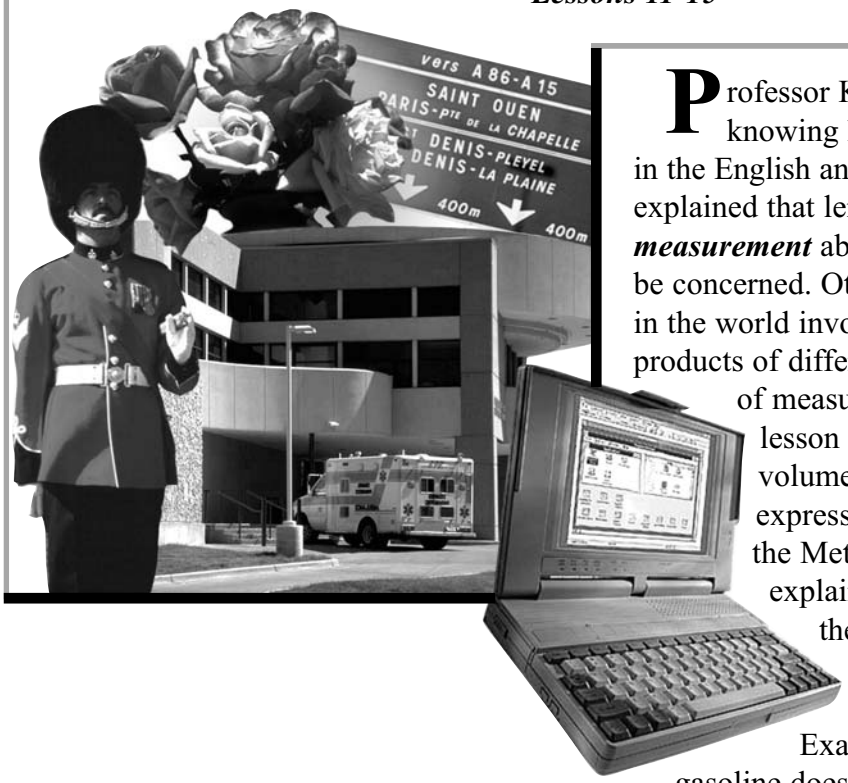
Lesson 15

*Converting*




## Section 3

***Lessons 11-15***




Professor K's children were excited about knowing how to convert units of length in the English and Metric Systems. Professor K explained that length was not the only ***measurement*** about which students needed to be concerned. Other important measurements in the world involve ***volume***. You buy and use products of different sizes, using different units of measurement everyday. In this lesson you will learn about units of volume. Units of volume can be expressed in the English System or the Metric System. This lesson will explain measurements of volume in the English System. Volume is the measure of the amount of space something occupies. Examples: How many gallons of gasoline does your gas tank hold? Professor K drank a pint of milk this morning.

**Table 15.1**

**Volume - English System**

| | | |
|---|---|---|
| 1 tablespoon, Tbsp, T | = | 3 teaspoons, tsp |
| 1 cup, c | = | 16 tablespoons, Tbsp |
| 1 pint, pt | = | 2 cups, c |
| 1 quart, qt | = | 2 pints, pts |
| 1 gallon, gal | = | 4 quarts, qts |

## VOCABULARY

***Measurement:*** size or quantity of something found by measuring; expressed in units
***Volume:*** the amount of space something occupies
***Convert:*** change from one type of measurement to another type of measurement

While looking at **Table 13.1** (in Lesson 13), note that when you change the units in the left column to units in the right column, multiplication solves the problem. When you ***convert*** units from the right column to the left column, division solves the problem. **Table 15.2** shows this procedure more clearly.

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

**Table 15.2**

| Multiply (larger to smaller) | | Divide (smaller to larger) | |
|---|---|---|---|
| Tbsp | → tsp | Tbsp | ← tsp |
| cup | → Tbsp | cup | ← Tbsp |
| pint | → cups | pint | ← cups |
| quart | → pints | quart | ← pints |
| gallon | → quarts | gallon | ← quarts |

The following examples should help clarify this procedure.

**Example Set 1**

| | | |
|---|---|---|
| **Step 1** | 32 cups = ?? gallon | To convert or change *32* cups into gallons, you must remember the conversion from **Table 15.1** that shows 2 cups = 1 pint. With this fact in mind, the problem of "How many gallons are in *32* cups?" is one of division. |
| **Step 2** | 32 ÷ 2 = 16 pints | To change *32* cups into gallons, you divide *32* cups by *2* cups. This means *32* cups = *16* pints. (See Step 2.) |
| **Step 3** | 16 ÷ 2 = 8 quarts | The next step is to divide *16* pints by *2* pints to see how many quarts are in *16* pints. After doing this, you determine the answer is *8* quarts. (See Step 3.) |
| **Step 4** | 8 ÷ 4 = 2 gallons | Next, to convert to gallons, you divide *8* quarts by *4* quarts. (See Step 4.) |

**Example Set 2**

| | | |
|---|---|---|
| **Step 1** | 16 quarts = ?? Tbsp | To convert quarts to tablespoons, once again you need to be familiar with the facts in **Table 15.1.** |
| **Step 2** | 16 quarts × 2 = 32 pints | Since *2* pints = 1 quart, you will multiply *16* quarts by *2*. Therefore, *16* quarts = *32* pints. (See Step 2.) |

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

| | |
|---|---|
| **Step 3** 32 pints × 2 = 64 cups | Now, since *2* cups = 1 pint, *32* pints will be multiplied by *2*. This means *32* pints = *64* cups. (See Step 3.) |
| **Step 4** 64 cups ×16 = 1,024 Tbsp | The final conversion is obtained by using the fact that 1 cup contains *16* tablespoons. You will have to multiply *64* cups by *16*. This gives you the final answer of *1,024* Tbsp. (See Step 4.) |

Professor K encourages you to complete the activity questions to demonstrate your knowledge of converting measurements of volume in the English System.



## LIFE PRINCIPLE



**"Your character determines how you measure up to emergencies."**

—Gene Hood

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.




©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

# *Basic Math Skills*

## Mathematics Charts

---

***LENGTH - ENGLISH SYSTEM***

| | | |
|---|---|---|
| 12 inches | = | 1 foot |
| 3 feet | = | 1 yard |
| 5.5 yards | = | 1 rod |
| 320 rods | = | 1 mile |
| 1,760 yards | = | 1 mile |
| 5,280 feet | = | 1 mile |

***VOLUME - ENGLISH SYSTEM***

| | | |
|---|---|---|
| 1 tablespoon, Tbsp, T | = | 3 teaspoons, tsp |
| 1 cup, c | = | 16 tablespoons, Tbsp, T |
| 1 pint, pt | = | 2 cups, c |
| 1 quart, qt | = | 2 pints, pts |
| 1 gallon, gal | = | 4 quarts, qts |

***WEIGHT - ENGLISH SYSTEM***

| | | |
|---|---|---|
| 1 pound, lb | = | 16 ounces, oz |
| 1 hundredweight, cwt | = | 100 pounds, lbs |
| 1 ton | = | 20 hundredweight, cwt |
| 1 ton | = | 2000 pounds, lbs |

***LENGTH - METRIC SYSTEM***

| | | |
|---|---|---|
| 1 meter | = | 1,000 millimeters |
| 1 meter | = | 100 centimeters |
| 1 kilometer | = | 1,000 meters |
| 1 centimeter | = | 10 millimeters |

***VOLUME - METRIC SYSTEM***

| | | |
|---|---|---|
| 1 liter | = | 1,000 milliliters |
| 1 liter | = | 100 centiliters |
| 1 kiloliter | = | 1,000 liters |
| 1 centiliter | = | 10 milliliters |

***WEIGHT - METRIC SYSTEM***

| | | |
|---|---|---|
| 1 gram | = | 1,000 milligrams |
| 1 gram | = | 100 centigrams |
| 1 kilogram | = | 1,000 grams |
| 1 centigram | = | 10 milligrams |

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

## *Mathematics Charts*

### *PERFECT SQUARES*

| | | | |
|---|---|---|---|
| $\sqrt{1}=1$ | $\sqrt{25}=5$ | $\sqrt{81}=9$ | $\sqrt{169}=13$ |
| $\sqrt{4}=2$ | $\sqrt{36}=6$ | $\sqrt{100}=10$ | $\sqrt{196}=14$ |
| $\sqrt{9}=3$ | $\sqrt{49}=7$ | $\sqrt{121}=11$ | $\sqrt{225}=15$ |
| $\sqrt{16}=4$ | $\sqrt{64}=8$ | $\sqrt{144}=12$ | $\sqrt{256}=16$ |

### *PERFECT CUBES*

| | | | |
|---|---|---|---|
| $\sqrt[3]{1}=1$ | $\sqrt[3]{-8}=-2$ | $\sqrt[3]{64}=4$ | $\sqrt[3]{-125}=-5$ |
| $\sqrt[3]{-1}=-1$ | $\sqrt[3]{27}=3$ | $\sqrt[3]{-64}=-4$ | $\sqrt[3]{216}=6$ |
| $\sqrt[3]{8}=2$ | $\sqrt[3]{-27}=-3$ | $\sqrt[3]{125}=5$ | $\sqrt[3]{-216}=-6$ |

$$P = \frac{\text{the number of elements in the set containing favorable outcomes}}{\text{the number of elements in the sample set}}$$

$$Midpoint = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

### *FORMULAS*

| Surface Area | Area | Volume | Perimeter/ Circumference | Others |
|---|---|---|---|---|
| **Sphere:** $A_s = 4\pi r^2$ | **Triangle:** $A = {}^1\!/_2 bh$ | **Sphere:** $V = {}^4\!/_3 \pi r^3$ | **Perimeter of a Rectangle:** $P = 2w + 2l$ | **Pythagorean Theorem:** $c^2 = a^2 + b^2$ |
| **Cylinder:** $A_s = 2\pi rh + 2\pi r^2$ | **Rectangle:** $A = l \times w$ | **Cone:** $V = {}^1\!/_3 \pi r^2 h$ | **Circumference of a Circle:** $C = 2\pi r$ or $C = \pi d$ | **Convert to Fahrenheit:** $F = {}^9\!/_5 C + 32$ |
| **Cube:** $A_s = 6s^2$ | **Circle:** $A = \pi r^2$ | **Rectangular Prism:** $V = l \times w \times h$ | | **Convert to Celsius:** $C = {}^5\!/_9 (F - 32)$ |
| **Lateral Surface Area of a Cone:** $A_{ls} = \pi rl$ | **Trapezoid:** $A = {}^1\!/_2 (b_1 + b_2)h$ | **Square Pyramid:** $V = ({}^1\!/_3) lwh$ | | **Distance:** $d = r \bullet t$ |
| | | **Triangular Prism:** $V = {}^1\!/_2 bhl$ | | **Slope:** $y = mx = b$ |
| | | **Cylinder:** $V = \pi r^2 h$ | | **Simple Interest:** $I = prt$ |

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

## *TIME CONVERSIONS*

1 week = 7 days
1 year = 12 months

## *CONVERSIONS*

| English to Metric | |
|---|---|
| 1 ounce (oz) | 28.35 grams (g) |
| 1 pound (lb) | 454 grams (g) |
| 1 ton (T) | 907.2 kilograms (kg) |

| Metric to English | |
|---|---|
| 1 gram (g) | .035 ounce (oz) |
| 1 kilogram (kg) | 2.2 pounds (lb) |
| 1 metric ton (mt) | 2200 pounds (lb) |

| English to Metric | |
|---|---|
| 1 cubic inch | 16.39 cubic cm |
| 1 gallon | 3.785 liters |
| 1 cubic foot | 28.316 liters |
| 1 quart | .946 liters |

| Metric to English | |
|---|---|
| 1 cubic cm | .061 cubic inch |
| 1 cubic meter | 35.315 cubic feet |
| 1 liter | 1.05 quarts |
| 1 liter | .264 gallon |

| English to Metric | |
|---|---|
| 1 inch | 2.54 cm |
| 1 foot | 0.3048 meter |
| 1 yard | 0.9144 meter |
| 1 mile | 1.6 km |

| Metric to English | |
|---|---|
| 1 centimeter | 0.3937 in. |
| 1 meter | 39.37 in. |
| 1 meter | 1.09 yds |
| 1 kilometer | 0.6213 mile |

## *AREA CONVERSIONS*

1 square centimeter = 0.155000 sq in.
1 square meter = 10.763910 sq ft
1 square inch = 645.16 sq mm
1 square foot = 0.09290304 sq meter
1 square yard = 0.836127 sq meter
1 square foot = 144 sq in.

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.




©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.